Online Appendix

Coordinated Effects in Merger Review

Simon Loertscher and Leslie M. Marx

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OA. Online Appendix

OA.1. Application

As described by Ivaldi et al. (2012), one industry in which coordinated effects and the potential for mavericks is a concern, and where price formation is, to a first-order approximation, characterized by efficient procurement, is the French audit industry.[1] Thus, the French audit industry provides an almost ideal application of the framework just presented. In addition, because large French firms are required to engage two auditors, the application allows us to illustrate the extension of the model to the case of multi-unit demand, which is detailed in Appendix OA.2.1.

Ivaldi et al. (2012) raise the questions of whether the French audit industry is at risk for coordination among the Big 4 firms and whether the fifth-largest firm, Mazars, should be viewed as a maverick, by which they mean "a firm with a drastically different cost structure, which is thus unwilling to participate to a collusive action" (Ivaldi et al., 2012, p. 40). Ultimately, they conclude that Mazars is not a maverick based on their qualitative and quantitative analysis, including econometric results indicating that Mazars is not properly viewed as a competitor with comparable capabilities to a Big 4 firm. Given that, they then conclude that the market *is* at risk for coordination by the Big 4 based on the *Airtours* criteria of sufficient transparency, the possibility of retaliation, and the absence of either a disruptive rival (that is, a maverick) or powerful buyers.[2]

We calibrate the model of second-price procurement to the data and then examine both whether the market is at risk for allocation by the Big 4 firms according to our framework and whether Mazars is a maverick according to our definition. In light of the characteristics of the French audit industry, including the feature that large companies must each hire two independent auditors (Ivaldi et al., 2012), we model each buyer as having value $v \geq \overline{c}$ for two-units, one from each of two different suppliers (and zero value for anything else), and as using an efficient procurement in its dominant strategy implementation.

To calibrate cost distributions, we use the data on 2006 revenue-based market shares for the top eight firms given in Ivaldi et al. (2012, Table 4.2). We assume that the data generating process is competitive bidding across a large number of procurements in which each bidder i's cost is an independent draw from a power-based distribution. It is then straightforward to write each supplier i's revenue share, $R_i / \sum_{j=1}^{n} R_j$, in terms of the parameters of the cost

[1] Ivaldi et al. (2012, pp. 79–80) describe the process of retaining an auditor as involving competitive bidding, which provides the foundation for a procurement model. In addition, they note that buyer power does not seem to be a strong force on the audit market (Ivaldi et al., 2012, p. 81), supporting the use of an efficient procurement model.

[2] CFI, 6 June 2002, case T-342/99, *Airtours v. Commission of the European Communities.*

distributions (for details, see Appendix OA.2.1). For identification, we assume that Ivaldi et al.'s "other" category consists of six symmetric suppliers (which ensures that they are among the smallest suppliers) and that the average cost parameter is equal to one, which pins down the level of the parameters.

Using the calibrated cost distributions, we can then calculate the critical shares and $\mathcal{I}^S$(Big 4), which are shown in Table OA.1. We find that $\mathcal{I}^S$(Big 4) is positive, which implies that, consistent with the findings of Ivaldi et al. (2012), the market is at risk for a market allocation among the Big 4 and that Mazars is not a maverick (because Mazars' presence in the market does not prevent the market from being at risk for allocation).

Table OA.1: Calibration and analysis of market allocation in the French audit industry

		Revenue-based shares	α_i	s_i(Big 4)	$\mathcal{I}^S$(Big 4)
Big 4	Ernst & Young	29.8%	3.8668	0.1289	0.5670
Big 4	Deloitte	21.4%	2.9934	0.1052	
Big 4	KPMG	22.2%	3.0820	0.1077	
Big 4	PWC	17.2%	2.5061	0.0913	
	Mazars	7.3%	1.1819		
	Grant Thornton	0.4%	0.0703		
	BDO	0.2%	0.0352		
	Constantin	0.3%	0.0528		
	6 others	0.2% each	0.0352		
	Total	100%	14		

To analyze the stability of a market allocation scheme, we also calculate the coordinated effects indices for various groups of participating suppliers, as shown in Table OA.2. As the table shows, participation by any three of the Big 4 is stable (the coordinated effects index for any three of the Big 4 is positive, but for any two is negative). The table also shows that the market is not at risk for pairwise allocation between any of the Big 4 firms (and the market is not at risk for pairwise allocation between Mazars and any one of the Big 4).

Overall our results are consistent with those of Ivaldi et al. (2012)—we find that the market is at risk for allocation by the Big 4 and that Mazars is not a maverick—but our analysis also points to the perhaps greater concern of allocation among subsets of three of the Big 4. Although the market is at risk for allocation by the Big 4, each individual member of the Big 4 would prefer not to participate in an allocation scheme if the alternative were for the remaining three firms to engage in a market allocation scheme. But for any subset of three of the Big 4 firms, each firm is pivotal to the feasibility of allocation.

Table OA.2: $\mathcal{I}^S(K)$ for various sets K of participating suppliers. Firms are: 1. Ernst & Young, 2. Deloitte, 3. KPMG, 4. PWC, 5. Mazars.

K	$\mathcal{I}^S(K)$	K	$\mathcal{I}^S(K)$	K	$\mathcal{I}^S(K)$
{1,2,3,4}	0.5670	{1,2}	−0.0691	{1,2,3,4,5}	0.6115
{1,2,3}	0.2905	{1,3}	−0.0640	{1,2,5}	0.0103
{1,2,4}	0.2852	{1,4}	−0.1135	{1,3,5}	0.0172
{1,3,4}	0.2852	{2,3}	−0.1044	{1,4,5}	−0.0567
{2,3,4}	0.2917	{2,4}	−0.1710	{2,3,5}	−0.0357
		{3,4}	−0.1617	{2,4,5}	−0.1420
				{3,4,5}	−0.1271

OA.2. Procurement setup with multi-unit suppliers and buyer power

Our definition of and test for coordinated effects generalize straightforwardly to an efficient procurement setup in which the buyer has multi-unit demand and suppliers have multi-unit capacities. (Without multi-unit demand, multi-unit capacities play no substantial role.) To be specific, we can allow the buyer to be characterized by a commonly known marginal value vector $\mathbf{v} = (v_1, ..., v_Q)$, where Q is the buyer's maximal demand, with $v_i \geq v_{i+1}$ for $i \in \{1, \dots, Q-1\}$ and for each supplier j to be characterized by a capacity κ_j and a vector of marginal costs $\mathbf{c}^j = (c_1^j, \dots, c_{\kappa_j}^j)$ satisfying $c_i^j \leq c_{i+1}^j$ for $i \in \{1, \dots, \kappa_j - 1\}$, where κ_j is (now) an integer. Assume that each supplier j's capacity κ_j is common knowledge, but that each supplier's marginal cost is its own private information. Assume also that for all j, $\mathbf{c}^j$ is distributed according to the commonly known, continuous distribution $G_j(\mathbf{c}^j)$ with support $[\underline{c}, \overline{c}]^{\kappa_j}$.

A simple and particularly convenient specification for the multivariate distribution $G_j(\mathbf{c}^j)$ is to assume that j's cost draw is the realization of κ_j independent, univariate random variables c drawn from the distribution $G_j(c)$. This implies that $G_j(\mathbf{c}^j)$ is given by the distribution if the κ_j-th order statistic from G_j. For example, the distribution of c_1^j is $G_{j,[1]}(c) = 1 - (1 - G_j(c))^{\kappa_j}$. Consequently, we refer to this as the *order statistics model*. This model also makes clear the sense in which the power-based parameterization captures a supplier's strength.

Following a merger between suppliers h and j, the merged entity's capacity is $\kappa_h + \kappa_j$. In the order statistics model, assuming pre-merger symmetry between j and h, so that $G_j = G_h = G$, the distribution of the minimum cost c_1^{hj} of the merged firm is $G_{hj,[1]}(c) = 1 - (1 - G(c))^{\kappa_h + \kappa_j}$.

The payoff (or revenue) equivalence theorems for multi-dimensional type spaces of Williams

(1999) and Krishna and Maenner (2001) imply that the generalized second-price auction with reserve prices for the m-th unit given by $\min\{v_m, \bar{c}\}$ is without loss of generality insofar as this is the profit-maximizing mechanism for the buyer subject to efficiency and individual rationality and incentive compatibility constraints for the suppliers. Consequently, the profit of every supplier j is pinned down by $\mathbf{v}$ and the distributions $(G_i(\mathbf{c}^i))_{i \in N}$ when all suppliers play their dominant strategies of reporting their types $\mathbf{c}^i$ truthfully.

Likewise, the expected profit $\Pi_i(K)$ when the suppliers $j \in K$ participate in a bidder allocation scheme when $i \in K$ is the designated bidder is pinned down by $G_i(\mathbf{c}^i)$ for $i \in K$ and $(G_h(\mathbf{c}^h))_{h \in N \setminus K}$. Consequently, $s_i(K) = \Pi_i / \Pi_i(K)$ as in the single-unit case, and the $\mathcal{I}^S(K)$ can be defined in the same way and with the same interpretation as before.

OA.2.1. Two-unit demand

With two-unit demand, $v \geq \bar{c}$, supplier i's expected revenue is the expected value of the second-lowest cost among the other $n-1$ suppliers, conditional on that cost being greater than supplier i's cost, and multiplied by the probability that supplier i's cost is one of the two lowest, which we can write as:

$$R_i = \int_{\underline{c}}^{\bar{c}} \int_{c}^{\bar{c}} y dH_i(y) dG_i(c),$$

where H_i is the distribution of the second-lowest cost among suppliers other than i. Similarly, supplier i's expected profit is

$$\Pi_i = \int_{\underline{c}}^{\bar{c}} \int_{c}^{\bar{c}} (y - c) dH_i(y) dG_i(c).$$

Letting H_K be the distribution of the second-lowest cost among suppliers not in set K, then for $i \in K$, supplier i's expected profit when suppliers in K coordinate and supplier i is selected to be the only member of K to bid is

$$\Pi_i(K) = \int_{\underline{c}}^{\bar{c}} \int_{c}^{\bar{c}} (y - c) dH_K(y) dG_i(c).$$

Now consider the power-based parameterization. Letting $A \equiv \sum_{k \in N} \alpha_k$ and $A_{-X} \equiv \sum_{k \in N \setminus X} \alpha_k$, we can write R_i, Π_i, and $\Pi_i(K)$ in terms of the parameters of the cost distributions as shown in the following lemma.

Lemma OA.1. *Assuming $G_i(c) = 1 - (1-c)^{\alpha_i}$ and $n \geq 3$, if the buyer has two-unit demand with $v \geq 1$, then supplier i trades with probability $q_i = \alpha_i \sum_{\ell \neq i} \left(\frac{1}{A_{-\{\ell\}}} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}} A} \right)$, has expected* *revenue*

$$R_i = \alpha_i \sum_{\ell \neq i} \left(\frac{\alpha_i + A_{-\{\ell\}}^2}{\left(1 + A_{-\{i,\ell\}}\right) A_{-\{\ell\}}(1 + A_{-\{\ell\}})} - \frac{A_{-\{i,\ell\}}\left(1 + \alpha_i\right)}{A_{-\{i\}}(1 + A_{-\{i\}}) A \left(1 + A\right)} \right),$$

and has expected profit $\Pi_i = R_i - C_i$*, where* C_i *is supplier* i*'s expected cost, given by*

$$C_i = \alpha_i \sum_{\ell \neq i} \left(\frac{1}{A_{-\{\ell\}}(1 + A_{-\{\ell\}})} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}} A(1 + A)} \right).$$

In addition, for $i \in K$*,* q_i*,* R_i *and* C_i *can be adjusted for the case of allocation by suppliers in* K *by summing over* $\ell \in N \backslash K$ *and letting* α_j *be zero for all* $j \in K \backslash \{i\}$*.*

Proof. Using the parameterization $G_i(c) = 1-(1-c)^{\alpha_i}$, for $n \geq 3$, the cdf of the second-lowest among the $n-1$ suppliers other than supplier i is

$$\begin{aligned} H_i(c) &\equiv 1 - \left(\prod_{j \neq i} (1 - G_j(c)) + \sum_{\ell \neq i} G_\ell(c) \times_{j \in N \backslash \{i,\ell\}} (1 - G_j(c)) \right) \\ &= 1 - \left((1-c)^{A_{-\{i\}}} + \sum_{\ell \neq i} (1 - (1-c)^{\alpha_\ell})(1-c)^{A_{-\{i,\ell\}}} \right), \end{aligned}$$

and the associated pdf is

$$h_i(c) = \sum_{\ell \neq i} (1 - (1-c)^{\alpha_\ell}) A_{-\{i,\ell\}} (1-c)^{A_{-\{i,\ell\}} - 1}.$$

The probability of trade for supplier i is

$$\begin{aligned} q_i &= \int_0^1 \int_c^1 \left(\sum_{\ell \neq i} (1 - (1-y)^{\alpha_\ell}) A_{-\{i,\ell\}} (1-y)^{A_{-\{i,\ell\}} - 1} \right) \alpha_i (1-c)^{\alpha_i - 1} dy dc \\ &= \int_0^1 \left(\sum_{\ell \neq i} A_{-\{i,\ell\}} \int_c^1 \left((1-y)^{A_{-\{i,\ell\}} - 1} - (1-y)^{A_{-\{i\}} - 1} \right) dy \right) \alpha_i (1-c)^{\alpha_i - 1} dc \\ &= \alpha_i \sum_{\ell \neq i} \left(\frac{1}{A_{-\{\ell\}}} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}} A} \right). \end{aligned}$$

So the market share of supplier i is $q_i/2$ (because the sum of all suppliers' probabilities of trade is 2 in the case of two-unit demand and no buyer power and $v \geq 1$).

Supplier i's expected revenue is

$$\begin{aligned} R_i &= \int_0^1 \int_c^1 y \left(\sum_{\ell \neq i} (1-(1-y)^{\alpha_\ell}) A_{-\{i,\ell\}} (1-y)^{A_{-\{i,\ell\}}-1} \right) \alpha_i (1-c)^{\alpha_i - 1} dy dc \\ &= \int_0^1 \left(\sum_{\ell \neq i} A_{-\{i,\ell\}} \int_c^1 \left(y(1-y)^{A_{-\{i,\ell\}}-1} - y(1-y)^{A_{-\{i\}}-1} \right) dy \right) \alpha_i (1-c)^{\alpha_i - 1} dc \\ &= \alpha_i \sum_{\ell \neq i} \left(\frac{1}{1+A_{-\{i,\ell\}}} \left(1 - \frac{A_{-\{i,\ell\}}}{A_{-\{\ell\}}(1+A_{-\{\ell\}})} \right) - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}}(1+A_{-\{i\}})A} \left(1 - \frac{A_{-\{i\}}}{1+A} \right) \right). \end{aligned}$$

Supplier i's expected cost is

$$\begin{aligned} C_i &= \int_0^1 \int_c^1 c \left(\sum_{\ell \neq i} (1-(1-y)^{\alpha_\ell}) A_{-\{i,\ell\}} (1-y)^{A_{-\{i,\ell\}}-1} \right) \alpha_i (1-c)^{\alpha_i - 1} dy dc \\ &= \int_0^1 c \left(\sum_{\ell \neq i} A_{-\{i,\ell\}} \int_c^1 \left((1-y)^{A_{-\{i,\ell\}}-1} - (1-y)^{A_{-\{i\}}-1} \right) dy \right) \alpha_i (1-c)^{\alpha_i - 1} dc \\ &= \alpha_i \sum_{\ell \neq i} \left(\frac{1}{A_{-\{\ell\}}(1+A_{-\{\ell\}})} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}} A (1+A)} \right). \end{aligned}$$

The remaining results follow by substitution and rearrangement. ■

OA.3. Applications with buyer power

In the U.S. DOJ's analysis of the proposed merger of oilfield services providers Halliburton and Baker Hughes, the agency identified the $400 million market of offshore cementing services as a relevant antitrust market.[3] According to the DOJ Complaint, the pre-merger market had essentially three suppliers: Halliburton, Baker Hughes, and Schlumberger.[4] Further, the information in the DOJ's complaint indicates that Halliburton and Baker Hughes had pre-merger market shares of 32% and 24% and that Schlumberger had a pre-merger market share of 43%, with the three suppliers accounting for 99% of the market.[5]

In this application, it seems reasonable to assume (as the merging parties argued) that the buyers, which include BP, Shell, and Exxon-Mobil, have buyer power. These buyers

[3] U.S. v. Halliburton Co. and Baker Hughes Inc., Complaint, Case 1:16-cv-00233-UNA, filed 6 April 2016 (DOJ Complaint).

[4] "In a strategic planning session, Halliburton's cementing executives recognized that this market is already a 'pure oligopoly' among the Big Three" (DOJ Complaint, p. 18).

[5] This can be deduced from the information provided in the DOJ Complaint that Schlumberger's market share was approximately 43%, the combined market share of Halliburton and Baker Hughes was approximately 56%, the pre-merger HHI was approximately 3500, and the post-merger HHI was approximately 5000. Although we can identify the shares of Halliburton and Baker Hughes as approximately 32% and 24%, it is not clear which supplier has the 32% share and which has the 24% share.

are large, sophisticated firms that purchased through competitive procurements. Thus, we calibrate distributions and calculate the coordinated effects index under the assumption of buyer power, but we also contrast the results with the case of no buyer power.

To facilitate the analysis of the case with buyer power, we use the parameterization $G_i(c) = c^{\alpha_i}$ (which implies linear virtual type functions), and we assume that v is sufficiently large that $v \geq \Gamma_i(\bar{c})$ for all i. As an identifying assumption, we assume that $\sum_{i=1}^{4} \alpha_i = 4$. Letting supplier 1 be Schlumberger and letting supplier 2 have market share 34% and supplier 3 have market share 24%, our calibration delivers $\alpha_1 = 0.0760$, $\alpha_2 = 0.0999$, $\alpha_3 = 0.1274$, and $\alpha_4 = 3.6967$. The calculation of the associated coordinated effects index for different sets of participants is shown in Table OA.3.

Pre-merger		Post-merger	
K	$\mathcal{I}^S(K)$	K	$\mathcal{I}^S(K)$
$\{1,2,3\}$	0.7617	$\{1,\mu_{2,3}\}$	0.6338
$\{1,2\}$	0.4625		
$\{1,3\}$	0.4198		
$\{2,3\}$	0.3780		

Table OA.3: Results for the oilfield services market of offshore cementing. Supplier 1 is Schlumberger, with pre-merger share 43%. Suppliers 2 and 3 are Halliburton and Baker Hughes (in unknown order), with pre-merger shares 34% and 24%. We denote the merger of Halliburton and Baker Hughes by $\mu_{2,3}$.

Holding fixed the distributions, without buyer power, we would have instead $\mathcal{I}^S(\{1,2,3\}) = 0.9510$ and $\mathcal{I}^S(\{1,\mu_{2,3}\}) = 0.8960$, which are larger than their corresponding values with buyer power, illustrating that the market would be at even greater risk, before and after the merger, if the buyers were not powerful.

As this shows, the market is at risk despite the presence of powerful buyers. And, holding fixed cost distributions, the market would be at greater risk if buyers were not powerful.

The market is also at risk for allocation by any pair of the suppliers in the Big 3.

OA.4. Generalization to multi-unit demand with buyer power

With buyer power, the main obstacle to the generalization to multi-unit supply and demand is that the optimal mechanism is not known when agents have multi-dimensional types. Even if one assumed single-unit suppliers in the pre-merger market, a merger would naturally lead to a multi-unit supplier.

However, all is not lost because there are circumstances in which even multi-unit buyers restrict themselves to buying at most one unit from each individual supplier. This may be due to (non-modelled) preferences for diversification, protection against further hold-up, or imposed by law (as in one of the applications in Section OA.1). Under these circumstances, all that matters for the buyer's optimal mechanism are the distributions of each seller j's lowest cost c_1^j, that is, $G_{j,[1]}(c_1^j)$, which is a one-dimensional variable. Hence, the standard mechanism design tools and results apply.

Let us briefly elaborate. The profit-maximizing mechanism for the buyer subject to incentive compatibility and individual rationality constraints given $n > Q$ is characterized as follows: For notational simplicity, let $c_j \equiv c_1^j$ and $G_j(c_j) \equiv G_{j,[1]}(c_1^j)$ with support $[\underline{c}, \overline{c}]$ and density $g_j(c_j)$ for all $j \in N$. Moreover, to simplify the analysis, assume as before that, for all $j \in N$, the virtual cost function $\Gamma_j(c_j)$ defined by

$$\Gamma_j(c_j) \equiv c_j + \frac{G_j(c_j)}{g_j(c_j)}$$

is increasing in c_j. Then, for a given realization $\mathbf{c} = (c_1, \ldots, c_n)$ and for given $\mathbf{v}$, the profit-maximizing mechanism for the buyer has the allocation rule of purchasing $m \in \{0, \ldots, Q\}$ units from the m suppliers with the lowest *virtual* costs, where, if $m < Q$, m is such that the m-th lowest virtual cost is less than v_m and the $m+1$-st lowest virtual cost exceeds v_{m+1}.

In the dominant strategy implementation of this mechanism, suppliers who do not produce receive (and make) no payments. Each supplier who trades is paid a threshold payment, that is, the highest cost that it could have reported without changing the fact that it trades. This pins down Π_i and $\Pi_i(K)$, and thereby $s_i(K)$ and $\mathcal{I}^S(K)$, just as in the single-unit case. For example, in the special case in which all suppliers are ex ante symmetric with $G_j = G$ for all j and thus $\Gamma_j = \Gamma$ for all j, the optimal mechanism can be implemented as via a second-price auction, in which the reserve price for the l-th unit is $\Gamma^{-1}(v_l)$. If the quantity traded is m, the m successful suppliers are paid $\min\{\Gamma^{-1}(v_m), c_{[m+1]}\}$, where $c_{[m+1]}$ denotes the $m+1$-st lowest cost.

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