

# Collusive Market Allocations\*

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## Abstract

Collusive schemes by suppliers often take the form of allocating customers or markets among cartel members. We analyze incentives for suppliers to initiate and sustain such a collusive schemes in a repeated procurement setting. We show that, contrary to some prevailing beliefs, staggered (versus synchronized) purchasing does not make collusion more difficult to sustain or initiate. Buyer defensive measures include synchronized rather than staggered purchasing, first-price rather than second-price auctions, more aggressive or secrete reserve prices, longer contract lengths, withholding information, and avoiding observable registration procedures. Inefficiency induced by defensive measures is an often unrecognized social cost of collusive conduct.

**Keywords:** synchronized vs staggered purchasing, sustainability and initiation of collusion, coordinated effects

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# 1 Introduction

Bid rigging is a widespread phenomenon that affects public and private procurement around the world, harming taxpayers and businesses.<sup>1</sup> It often takes the form of a customer or market allocation, defined by the U.S. Department of Justice as:

**Customer or Market Allocation:** In this scheme, co-conspirators agree to divide up customers or geographic areas. The result is that the co-conspirators will not bid or will submit only complementary bids when a solicitation for bids is made by a customer or in an area not assigned to them. This scheme is most commonly found in the service sector and may involve quoted prices for services as opposed to bids. (USDOJ, 2015, p. 3)

For example, in the cartel in Industrial Tubes, “The participants agreed also upon allocation of key customers and volumes supplied to them.... The customer allocation was also implemented by quoting artificially high prices, if a supplier was approached by a customer that was not allocated to it.”<sup>2</sup> In the French Bakers cartel, the market allocation was defined in geographic terms, in some cases in great detail.<sup>3</sup> Customer or market allocations might be initiated by a supplier withdrawing from one of the markets, or submitting the maximum allowed bid in that market.<sup>4</sup>

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<sup>1</sup>Given the widespread use of competitive tenders in business-to-business transactions, any effective collusion in intermediate goods markets frequently requires an element of bid rigging.

<sup>2</sup>EC decision in *Industrial Tubes* (paras. 106–107), <http://ec.europa.eu/competition/cartels/cases/cases.html>. Other examples include the EC decisions in Choline Chloride, Copper Plumbing Tubes, Electrical and Mechanical Carbon and Graphite Products, Food Flavour Enhancers, Industrial and Medical Gases, Industrial Bags, Methylglucamine, Monochloroacetic Acid, and Zinc Phosphate (Marshall and Marx, 2012, Table 6.1). For a discussion of the legal treatment of market and customer allocations, see Marshall and Marx (2012, p. 123, footnote 43).

<sup>3</sup>“Each member’s area were delimited on captioned road maps, dated and signed by the members. These areas were determined very precisely, sometimes down to the street” (Autorité de la Concurrence, Press Release, “Distribution of Raw Materials and Equipment to Bakers,” 8 July 2019, p. 1, [http://www.autoritedelaconcurrence.fr/user/standard.php?lang=fr&id\\_rub=697&id\\_article=3454](http://www.autoritedelaconcurrence.fr/user/standard.php?lang=fr&id_rub=697&id_article=3454)). In another recent example, suppliers of corn tortillas in Mexico established minimum distances between tortillerías and an obligation to respect agreed sales areas (COFECE Press Release, “Sanciona COFECE a 5 personas físicas y a 3 asociaciones tortilleras por pactar precios y segmentar el mercado de tortilla de maíz en Palenque, Chiapas,” July 22, 2019, <https://www.cofece.mx/sanciona-cofece-a-tortilleros-por-pactar-precios-en-palenque/>).

<sup>4</sup>In *U.S. v. Champion International Corp.* (1975 U.S. Dist. Or.), bidders in Forest Service timber auctions apparently initiated a division of auctions. Following a period of aggressive bidding, “This ‘bidding war’ came to a sudden end on June 2, 1967, when defendant Vernon Morgan ‘was surprised’ to find no one bidding against him at an auction of a small offering of government timber. Morgan decided, he said, ‘to experiment’, and later that same day he offered no bid against defendant Freres on another sale, with the result that Freres took the second sale at a nominal figure over the appraised price” (Justia US Law, <https://law.justia.com/cases/federal/appellate-courts/F2/557/1270/272936/>).

Colluding suppliers engaged in, or attempting to initiate, a customer or market allocation are affected by the timing with which buyers procure their inputs. For example, it may be that all buyers procure inputs twice a year by holding procurements in January and July (synchronous purchases), or that some make their purchases in January and July, while others purchase in April and October (staggered purchases). Interestingly, in the context of the AT&T–Time Warner merger, the judge found that “the *staggered*, lengthy industry contracts would make [coordination] extremely risky” (emphasis added) because one party would have to “jump first” on the hope that the other would do the same later on, concluding that “putting such blind faith in one’s chief competitor strikes this Court as exceedingly implausible!” (Leon, 2018, p. 163).<sup>5</sup>

To study the impact of purchasing patterns on the scope for collusion based on a market allocation, we consider a repeated procurement setting in which tenders are either synchronized or staggered. In contrast with prior formal analyses, and prompted by the above legal opinion, we also address the *initiation* of such collusion. We show that the initiation of a market allocation is profitable whenever it is sustainable, and that staggered contracts actually facilitate collusion. In the same way that “short contracts do not induce firms to deviate from the cartel since punishment would be inflicted in the near future” (Albano et al., 2006a, p. 352), holding fixed the contract length, staggered contracting reduces the time before punishment may arrive, making collusion easier to sustain and more profitable to initiate.

This implies that buyers facing the threat of collusion may be able to deter or destabilize collusion by using synchronized rather than staggered purchasing. In addition, we show that other defensive measures by buyers include using first-price rather than second-price auctions, setting more aggressive reserves or secret reserves, negotiating longer contracts, withholding information on placed bids, and avoiding observable registration procedures. Some of these buyer defensive measures increase inefficiency, which is an often unrecognized social cost of collusive conduct. Moreover, some defensive measures may be sufficiently costly to the buyer that the buyer prefers to accommodate rather than deter collusion.

The economics literature has studied the use of various collusive schemes, including the market and customer allocations that are the focus of our paper (see, e.g., Harrington, 2006; Marshall and Marx, 2012).<sup>6</sup> Indeed, Stigler (1964, p. 46) recognizes the effectiveness

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<sup>5</sup>The role of staggered contract renewal schedules in buttressing market power has been a concern for antitrust. As described by Cabral (2017), Pullman Co. staggered contracts with railroads for sleeping car services were viewed by the U.S. Department of Justice as contributing to Pullman’s monopoly power (*U.S. v. Pullman Co.*, Civil Action No. 994, 50 F.Supp. 123 (1943)). As described by Jing and Winter (2012), Nielsen used staggered contracts with Canadian grocery chains to erect a barrier to entry to other providers of market tracking services. We focus on collusion rather than on entry barriers.

<sup>6</sup>Rey and Stiglitz (1995) show that exclusive territories, which are a type of market allocation, can

of customer allocations as a collusive scheme when he notes that, relative to fixing market shares, “Almost as efficient a method of eliminating secret price-cutting is to assign each buyer to a single seller.” In related work on market allocations in a repeated game setup, Byford and Gans (2014) compare collusion within versus across markets. We compare instead different purchasing patterns (staggered versus synchronized) and different auction formats (first-price versus second-price).

As already mentioned, the literature has focused on the sustainability of collusion, with much less attention devoted to its initiation. For example, folk theorem style results do not address the process by which firms coordinate on strategies that deliver monopoly profits. As stated by Ivaldi et al. (2003, p. 6), “While economic theory provides many insights on the nature of tacitly collusive conducts, it says little on how a particular industry will or will not coordinate on a collusive equilibrium, and on which one.”<sup>7</sup> The literature on collusive price leadership, however, provides examples in which one firm leads the announcement of collusive price increases, in some cases with a future effective date to allow for time to retract the price increase should its rivals choose not to follow (Marshall et al., 2008). In the setup we consider, even if the suppliers can “reason their way” to a market allocation (see Green et al., 2015), one supplier must necessarily “go first” by withdrawing from the rival’s market.<sup>8</sup> This allows us to study the incentives to initiate a market allocation through a unilateral decision to stop competing for one of the buyers.

Given our focus on purchasing patterns, our work is related to that of Dana and Fong (2011).<sup>9</sup> They consider a model with overlapping generations of price-taking customers and show that longer contracts, which create a staggered pattern, make tacit collusion easier to sustain because it reduces the number of customers on which a firm can deviate at any point in time. In contrast, we consider long-lived customers that conduct competitive procurements and focus on synchronous versus staggered contracts of fixed length (although we discuss the impact of contract length as well). We moreover address the

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reduce competition.

<sup>7</sup>This has sometimes led to the suggestion that explicit agreement might be required. For example, Green and Porter (1984, footnote 5) state, “It is logically possible for this agreement to be a tacit one which arises spontaneously. Nevertheless, in view of the relative complexity of the conduct to be specified by this particular equilibrium and of the need for close coordination among its participants, it seems natural to assume here that the equilibrium arises from an explicit agreement.”

<sup>8</sup>Documents uncovered in the investigation of the *Industrial Tubes* cartel to include the following meeting notes: “- General increase; - Customer by customer; - LMI [Europa Metalli-LMI S.p.A.] is ready for discussion; - PREPARE A CUSTOMER LIST” and “We will withdraw from [redacted] with regard to smooth tubes (will continue efforts in IGT [inner grooved tubes])” (EC Decision in *Industrial Tubes*, paras. 126 and 137).

<sup>9</sup>In a contribution game setting, Admati and Perry (1991) show that agents are unable to overcome free-riding when their moves are staggered over time. In contrast, Marx and Matthews (2000) show that sufficiently patient agents can overcome free-riding when the possibility of simultaneous moves is introduced.

initiation as well as the sustainability of collusion, and the assumption of strategic purchasers enables us to analyze buyer defensive measures. Relatedly, Albano and Spagnolo (2005) contrast sealed-bid auctions that use a simultaneous versus sequential format and show that the sequential format facilitates collusion because deviations can be identified and reacted to within the context of the auction. In a non-collusive setting, Cabral (2017) shows that the staggering of contracts can benefit an incumbent by deterring entry by a rival in the presence of economies of scale. Further, Iossa et al. (2019) find that the profit gain from moving a duopoly to a monopoly is greater when contracts are staggered, but they do not analyze the effects on collusion.

Finally, our work touches on a number of other issues that have foundations in prior literature. Regarding contract length, Iossa and Rey (2014) analyze the impact of contract length on incentives for providing quality. Related to buyer defensive measures against bidding rings, our findings on the benefits of first-price auctions, reduced information disclosure, and not requiring advance bidder registration are broadly consistent with the literature (e.g., Kovacic et al., 2006; Marshall and Marx, 2009; Kumar et al., 2015; Marshall et al., 2014; Marx, 2017). On the challenges faced by firms trying to initiate collusion, particularly in the absence of communication, see Green et al. (2015).

The remainder of the paper is organized as follows. Section 2 presents the setup. In Section 3, we analyze the sustainability and initiation of a market allocation. Section 4 considers defensive measures by buyers facing the threat of market allocation by their suppliers, including strategies revolving around reserves, contract duration, and information disclosure. In Section 5, we provide several extensions: we show that the model can be reinterpreted in terms of Bertrand competition, that key results are robust to alternative information and cost assumptions and to the possibility of asymmetric reserves, and that supplier registration increases vulnerability to collusion. In Section 6, we conclude.

## 2 Setup

We are interested in firms' ability to collude by allocating markets among themselves, and in the factors that may affect their incentives to initiate and sustain such collusion. Of particular interest are the timing of purchasing (synchronous versus staggered procurements) and the auction format (first-price versus second-price auctions). To this aim, we consider a discrete-time, infinite-horizon setting with two buyers that operate in separate markets. Because each buyer corresponds to its own market, we use the terms buyer and market interchangeably. One may alternatively use the interpretation that there is a single buyer holding procurements for two distinct customer segments, products or services, or geographic areas. We return to this interpretation in Section 4.

- *Supply.* There are two suppliers who, at the beginning of each period, draw their costs of serving any market in that period, where the cost distribution,  $G$ , has finite, positive density  $g$  over the support  $[\underline{c}, \bar{c}]$ . Whether a supplier's cost is the same for both markets or whether it is market specific does not affect the analysis. For the sake of exposition, we adopt the former interpretation. Cost draws are independent across suppliers and time, and are the suppliers' private information.

- *Demand.* Regarding purchasing patterns, we assume that, every other period, each buyer wishes to make a purchase, for which it has value  $v > \underline{c}$ . We contrast the cases of synchronized and staggered purchasing patterns. In the case of synchronized purchasing, both buyers hold procurements in the same periods (e.g., odd-numbered ones). In the case of staggered purchasing, one buyer holds procurements in odd-numbered periods and the other buyer holds procurements in even-numbered periods. We assume that suppliers bid simultaneously within a given procurement and across procurements held in the same period.

As for auction formats, we consider both second-price and first-price auctions, with a reserve equal to  $r \in (\underline{c}, \bar{c}]$ ; reserves outside this range are dominated for the buyer. In a one-shot setting, the Bayes Nash equilibrium is unique for the first-price auction (Lebrun, 1999), and for the second-price auction, it is unique if  $r < \bar{c}$  (Blume and Heidhues, 2004),<sup>10</sup> in which case—from the payoff equivalence theorem—it yields the same payoffs as a first-price auction. For the case of a second-price auction with  $r = \bar{c}$ , we focus on the equilibrium that is the limit of the unique equilibria as  $r$  approaches  $\bar{c}$  from below. Alternatively, one can assume that the buyer can always credibly cap the price it pays an amount that is arbitrarily close to but below  $\bar{c}$ . For the purposes of Section 3, we take the reserve as given. In Section 4, we extend the analysis by studying buyers' strategic choices of their reserve prices.

All agents are risk neutral with quasi-linear utility and discount the future according to the common discount factor  $\delta \in [0, 1)$ . All of the above is common knowledge.

- *Monopoly and competitive benchmarks.* Let

$$\pi^m(c) \equiv \max\{0, r - c\} \tag{1}$$

denote a supplier's monopoly payoff per market given cost  $c$ , and let

$$\bar{\pi}^m \equiv \mathbb{E}_c[\pi^m(c)] = \int_{\underline{c}}^r G(c)dc \tag{2}$$

denote a supplier's expected monopoly payoff per market.

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<sup>10</sup>For  $r \geq \bar{c}$ , there is a continuum of Bayes Nash equilibria even in a one-shot second-price auction.

Similarly, let  $\pi^c(c)$  denote the expected competitive payoff per market for a supplier with cost  $c$ , that is,

$$\pi^c(c) \equiv \mathbb{E}_{\tilde{c}} [\max\{0, \min\{r, \tilde{c}\} - c\}], \quad (3)$$

where  $\tilde{c}$  denotes the rival's cost, and let

$$\bar{\pi}^c \equiv \mathbb{E}_c [\pi^c(c)] = \int_{\underline{c}}^r G(c) [1 - G(c)] dc \quad (4)$$

denote a supplier's expected competitive payoff per market. As is clear from these expressions, competitive payoffs are lower than monopoly payoffs.

Figure 1 displays the suppliers' payoff and expected payoff functions when costs are uniformly distributed over  $[0, 1]$ .

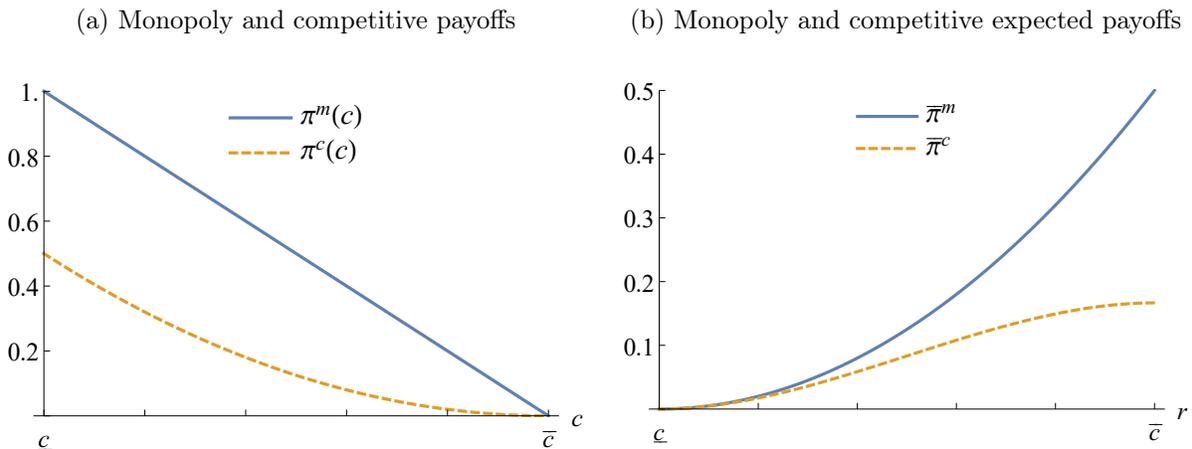


Figure 1: Expected payoffs under monopoly and competition when costs are uniformly distributed over  $[0, 1]$ . Panel (a) assumes  $r = \bar{c}$  and varies  $c$  from  $\underline{c}$  to  $\bar{c}$ . Panel (b) takes expectations over  $c$  as  $r$  varies from  $\underline{c}$  to  $\bar{c}$ .

*Remark: on the timing of suppliers' costs.* The baseline setup implicitly assumes that a supplier's cost of meeting the needs of one market in period  $t$ ,  $c^t$ , is entirely determined in period  $t$ . One interpretation is that the buyer purchases what it needs for two periods and manages the inventories. Another possible interpretation is that the cost of fulfilling the contractual obligations in period  $t$  is given by  $c^t$  and observed by the supplier at the beginning of that period. According to this interpretation, a supplier's expected cost of meeting one market's needs at periods  $t$  and  $t + 1$  is

$$C^t \equiv c^t + \delta \mathbb{E}_{c^{t+1}} [c^{t+1}].$$

Let  $R$  denote the reserve price in a market for a two-period contract and define

$$r \equiv R - \delta \mathbb{E}_c [c]$$

as the implicit reserve price for the first of the two periods. For a supplier with cost  $C^t$ , the monopoly profit is equal to

$$\max\{0, R - C^t\} = \max\{0, r - c^t\} = \pi^m(c^t),$$

and the expected competitive payoff is equal to

$$\mathbb{E}_{\tilde{C}^t} \left[ \max\{0, \min\{R, \tilde{C}^t\} - C^t\} \right] = \mathbb{E}_{\tilde{c}^t} \left[ \max\{0, \min\{r, \tilde{c}^t\} - c^t\} \right] = \pi^c(c^t).$$

The suppliers' profits are therefore the same as before. Consequently, our analysis applies to both interpretations of the suppliers' costs.

### 3 Market allocation

A market allocation scheme assigns each market to a designated supplier and specifies that the other supplier should not bid less than the reserve in that market. When the reserve is less than  $\bar{c}$ , it is, however, both efficient and profitable to allow the other supplier to trade at the reserve when the designated supplier's own cost exceeds the reserve. In addition, in a second-price auction, the gains from deviations are lowest when the designated supplier bids at cost. Hence, in what follows, we consider the following market allocation scheme: (i) the designated supplier always bids at cost in a second-price auction, whereas in a first-price auction, it bids slightly below the reserve when its cost lies below it, and at cost otherwise; and (ii) the non-designated supplier bids the reserve whenever its cost lies below it, and bids at cost otherwise.<sup>11</sup> We assume that there are no direct transfers between the suppliers and that bids are observed before the next procurement.<sup>12</sup> When considering attempts by suppliers to signal the initiation of collusion, we focus on signaling by bidding the reserve, as this provides a credible signal of the initiator's intent.

We begin in Section 3.1 by discussing the profitability of a market allocation. Then in Section 3.2 we study the sustainability and in Section 3.3 the initiation of a market allocation. Finally, in Section 3.4 we show that our insights readily apply to sales auctions

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<sup>11</sup>Specifying that the non-designated supplier should not bid at all (or, equivalently, bid above the reserve) would reduce expected profits, which also makes collusion more difficult to sustain, but would not affect our insights about the timing of purchases or the auction format. For further analysis of profitable tacit collusive schemes without explicit communication, see Skrzypacz and Hopenhayn (2004).

<sup>12</sup>We relax the assumption of observable bids in Section 4.3.

as well.

### 3.1 Profitability

Under a market allocation, the designated supplier obtains the monopoly profit and the other supplier obtains an expected payoff equal to

$$\pi^n(c) \equiv [1 - G(r)] \pi^m(c),$$

which accounts for the probability  $1 - G(r)$  that the designated supplier's cost exceeds  $r$ . The non-designated supplier's expected payoff is therefore

$$\bar{\pi}^n \equiv \mathbb{E}_c [\pi^n(c)] = [1 - G(r)] \bar{\pi}^m.$$

Observe that

$$\bar{\pi}^n = \int_{\underline{c}}^r [1 - G(r)] G(c) dc < \int_{\underline{c}}^r [1 - G(c)] G(c) dc = \bar{\pi}^c.$$

The market allocation is therefore profitable for the suppliers in expectation if and only if:<sup>13</sup>

$$\bar{\pi}^m + \bar{\pi}^n > 2\bar{\pi}^c. \quad (5)$$

Thus, we say that a market is “at risk” for a market allocation if (5) holds,<sup>14</sup> and otherwise that it is “not at risk.” We then have the following result:<sup>15</sup>

**Lemma 1.** *The market is always at risk for a market allocation if the distribution  $G$  has a monotone reverse hazard rate (i.e., if  $g(c)/G(c)$  is decreasing in  $c$ ). Furthermore, when  $r = \bar{c}$ , a market is at risk if and only if*

$$\bar{c} - \mathbb{E}_c [c] > \mathbb{E}_{c_1, c_2} [\max\{c_1, c_2\} - \min\{c_1, c_2\}]. \quad (6)$$

*Proof.* See Appendix A.1.

It follows from Lemma 1 that a market is not always at risk because condition (6)

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<sup>13</sup>Although we focus on symmetric suppliers, our results are not knife edged in the sense that asymmetries between the suppliers' distributions can be accommodated. For example, for asymmetric suppliers, (5) becomes  $\min\{\bar{\pi}_1^m + \bar{\pi}_2^n, \bar{\pi}_2^m + \bar{\pi}_1^n\} > \bar{\pi}_1^c + \bar{\pi}_2^c$ , where, for  $k \neq h \in \{1, 2\}$ ,  $\bar{\pi}_k^m = \int_{\underline{c}}^r G_k(c) dc$ ,  $\bar{\pi}_h^n = [1 - G_k(r)] \bar{\pi}_h^m$ , and  $\bar{\pi}_k^c = \int_{\underline{c}}^r [1 - G_h(c)] G_k(c) dc$ .

<sup>14</sup>The expression in (5) can also be written as  $\int_{\underline{c}}^r [2G(c) - G(r)] G(c) dc > 0$ .

<sup>15</sup>Lemma 1 relates to the condition in Loertscher and Marx (2019a) for a market to be at risk for coordinated effects.

is satisfied for some but not all distributions.<sup>16</sup> However, a market is at risk whenever the distribution  $G(c)$  is log-concave, that is, satisfies  $[\ln G(c)]'' < 0$ .<sup>17</sup> The family of distributions with this property is large and includes most of the “standard” distributions such as the uniform, normal, exponential, power, and extreme value distribution.<sup>18</sup>

In what follows, we analyze the critical discount factor required to support a market allocation when a market is at risk. For the sake of exposition, we assume that any deviation results in competitive conduct thereafter.

### 3.2 Sustainability

We first derive the collusive and deviation payoffs, beginning with the case of synchronized purchasing. Consider a period in which both buyers hold procurements. Given its cost realization  $c$ , a supplier has an expected payoff from cooperation of

$$\Pi^{Sync}(c) \equiv \pi^m(c) + \pi^n(c) + \frac{\delta^2}{1 - \delta^2}(\bar{\pi}^m + \bar{\pi}^n).$$

Obviously, a supplier cannot gain from deviating when its cost exceeds the reserve. When instead the supplier’s cost  $c$  lies below  $r$ , in a second-price auction the optimal deviation consists of bidding at cost, and it generates an expected profit equal to the competitive profit:

$$\pi_{SPA}^d(c) \equiv \pi^c(c).$$

In a first-price auction, the optimal deviation consists instead of slightly undercutting the designated supplier’s target price, and it yields an expected profit equal to the monopoly profit:

$$\pi_{FPA}^d(c) \equiv \pi^m(c).$$

Because the designated supplier bids the reserve in first-price auctions, and bids instead at cost in second-price auctions, the gains from deviating are higher in the former case:  $\pi_{FPA}^d(c) > \pi_{SPA}^d(c)$ .

The deviation triggers reversion to competitive bidding in both markets thereafter. Hence, given the auction format  $a \in \{SPA, FPA\}$ , the deviation yields a total expected payoff of

$$\tilde{\Pi}_a^{Sync}(c) \equiv \pi^m(c) + \pi_a^d(c) + \frac{\delta^2}{1 - \delta^2}2\bar{\pi}^c.$$

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<sup>16</sup>For example, (6) is not satisfied when  $[c, \bar{c}] = [0, 1]$  and  $g(x) = 0.05$  for  $x \in [0, 0.9]$ , whereas  $g(x) = 9.55$  for  $x \in (.9, 1]$ . This distribution thus has a long left tail and high probability close to the upper bound of the support; we then have  $\bar{c} - \mathbb{E}_c[c] = 0.0725 < \mathbb{E}_{c_1, c_2}[c_{(2)} - c_{(1)}] = 0.0740$ .

<sup>17</sup>To see this, notice that  $[\ln G(c)]' = g(c)/G(c)$ . Hence, log-concavity of  $G(c)$  implies that  $g(c)/G(c)$  is decreasing.

<sup>18</sup>See Bagnoli and Bergstrom (2005, Table 1) for a more comprehensive list.

We now examine the contrasting case of staggered purchasing, in which one buyer purchases in odd periods and the other buyer purchases in even periods. In any given period, the designated supplier has no incentive to deviate. Given its cost realization  $c$ , the other supplier's expected payoff from participation in the market allocation is

$$\Pi^{Stag}(c) \equiv \pi^n(c) + \frac{\delta}{1-\delta^2}\bar{\pi}^m + \frac{\delta^2}{1-\delta^2}\bar{\pi}^n.$$

Given the auction format  $a$ , the most profitable deviation yields again an expected profit  $\pi_a^d(c)$  in the current period and triggers a price war forever. Hence, the expected payoff from the deviation is

$$\tilde{\Pi}_a^{Stag}(c) \equiv \pi_a^d(c) + \frac{\delta}{1-\delta}\bar{\pi}^c.$$

Collusion is sustainable when suppliers have no incentives to deviate, that is, for  $s \in \{Sync, Stag\}$ , when  $\Pi^s(c) \geq \tilde{\Pi}_a^s(c)$  for every cost realization  $c$ . These sustainability conditions are satisfied when the loss from reverting to competition in the future (which thus does not depend on the current cost  $c$ ) offsets the short-term gains from a deviation,  $\pi_a^d(c) - \pi^n(c)$ . Because these gains decrease with the supplier's cost,<sup>19</sup> the most stringent sustainability condition can be expressed as:

$$L^s(\delta) \geq S_a, \tag{7}$$

where

$$S_a \equiv \pi_a^d(\underline{c}) - \pi^n(\underline{c}) \tag{8}$$

represents the *short-term stake*, namely, the gain from a deviation for a supplier that has the lowest possible cost,  $\underline{c}$ , whereas  $L^s(\delta)$  represents the *long-term stake*, namely, the difference between a supplier's expected profit from collusion and from competition (i.e., punishment) in all future periods.

The short-term stake  $S_a$  depends only on the auction format and is higher for first-price auctions:

$$S_{FPA} - S_{SPA} = \pi^m(\underline{c}) - \pi^c(\underline{c}) > 0. \tag{9}$$

By contrast, the long-term stake  $L^s$  depends only on the purchasing pattern: for synchronized purchasing, it is equal to

$$L^{Sync}(\delta) \equiv \frac{\delta^2}{1-\delta^2}(\bar{\pi}^m + \bar{\pi}^n - 2\bar{\pi}^c), \tag{10}$$

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<sup>19</sup>For  $c < r$ , the derivative of  $\pi_a^d(c) - \pi^n(c)$  with respect to  $c$  is equal to  $-G(r) < 0$  for first-price auctions and to  $G(c) - G(r) < 0$  for second-price auctions.

whereas for staggered purchasing, it is equal to

$$L^{Stag}(\delta) \equiv \frac{\delta}{1-\delta} \left( \frac{\bar{\pi}^m + \delta \bar{\pi}^n}{1+\delta} - \bar{\pi}^c \right). \quad (11)$$

From (5), in both cases the long-term stake  $L^s$  is positive and increases from 0 to infinity as  $\delta$  increases from 0 to 1.<sup>20</sup> It follows that there exists a threshold for  $\delta$  above which collusion is sustainable, and below which it is not. Furthermore, the long-term stake is higher for staggered purchasing:

$$L^{Stag}(\delta) - L^{Sync}(\delta) = \frac{\delta}{1+\delta} (\bar{\pi}^m - \bar{\pi}^c) > 0. \quad (12)$$

It follows directly from (9) and (12) that:

**Proposition 1.** *Collusion is easier to sustain under staggered purchasing than under synchronized purchasing, and with second-price auctions than with first-price auctions.*

The intuition is simple. Under synchronous purchasing, a deviation cannot be punished until the renewal of both contracts, which occurs two periods later. By contrast, under staggered purchasing, a deviation by the non-designated supplier can be punished in the next period, when that supplier's own market comes up for renewal. That second-price auctions facilitate collusion follows from the fact that, as noted above, they limit the gains from deviations by allowing the designated supplier to continue to use the competitive strategy of bidding at cost.<sup>21</sup>

This intuition suggests that the insights are quite robust. For example, they remain valid when the non-designated supplier completely withdraws from the market, regardless of its cost. Although such withdrawal is less efficient than the market allocation scheme considered so far (see footnote 11), staggered purchasing and second-price auctions still facilitate collusion, compared with synchronous purchasing and first-price auctions. In Section 4.3, we show further that the insights regarding the purchasing pattern carry over when bids are not observed (and/or the price paid to the winning supplier is not disclosed).

Because the long-term stake increases with  $\delta$ , the critical discount factor for sustainability is the value of  $\delta$  that solves (7) with equality. For synchronized purchasing, these

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<sup>20</sup>This is obvious for  $L^{Sync}(\delta)$ , and then follows from (12) for  $L^{Stag}(\delta)$ .

<sup>21</sup>In the context of wholesale electricity markets, Fabra (2003) shows that uniform auctions (giving all selected suppliers the same market-balancing price) facilitate collusion compared to discriminatory auctions that pay each selected supplier its own bid price. The reason is that, with uniform auctions, a few suppliers submitting high bids suffices to drive the balancing price to the desired level; as here with second-price auctions, the other firms can still bid at cost.

threshold discount factors are characterized by

$$\frac{\delta^2}{1 - \delta^2} = \frac{S_a}{\bar{\pi}^m + \bar{\pi}^n - 2\bar{\pi}^c}, \quad (13)$$

and for staggered purchasing by

$$\frac{\delta}{1 - \delta^2} = \frac{S_a}{\bar{\pi}^m + \delta\bar{\pi}^n - (1 + \delta)\bar{\pi}^c}. \quad (14)$$

### 3.3 Initiation

We now study the incentives to initiate a market allocation through a unilateral decision to stop competing for one of the buyers. A complete withdrawal may fail to signal the seller's willingness to initiate a market allocation (unless sellers observe each other's costs) because it is consistent with competitive bidding in the event that the supplier, facing a cost exceeding the reserve, cannot be profitably active anyway. By contrast, when its cost lies below the reserve, the initiator can credibly signal its intention by bidding the reserve. In this way, the initiator signals that it is prepared to supply at that price if the other supplier's cost happens to exceed the reserve, implying that it could have chosen to compete, but decided instead to confer market power to the other supplier.

Assuming that collusion is sustainable, bidding the reserve to initiate it involves a short-term sacrifice but increases future profits. Interestingly, the long-term stake, which amounts to switching from competition to collusion in every future tender, is the same as for sustainability. For synchronous purchasing, it is again given by (10). Likewise, for staggered purchasing (assuming that, having assigned the current tender to the other supplier, the initiator becomes the designated seller in the next tender), the long-term stake is, as before, given by (11).

The short-term sacrifice amounts to foregoing the competitive profit and obtaining instead the profit expected from bidding the reserve when the other supplier bids competitively; this sacrifice is therefore equal to  $\pi^c(c) - \pi^n(c)$ , which decreases with the initiator's cost. It follows that initiation is profitable whenever it is profitable for the lowest possible cost, which amounts to:

$$L^s(\delta) \geq \hat{S} \equiv \pi^c(\underline{c}) - \pi^n(\underline{c}). \quad (15)$$

Comparing (7) and (15) and noting that

$$\hat{S} = S_{SPA} < S_{FPA}$$

establishes the following proposition:

**Proposition 2.** *Under second-price auctions, initiating collusion (by bidding the reserve in one market) is profitable if and only if collusion is sustainable. Under first-price auctions, initiating collusion is strictly profitable whenever it is sustainable.*

The intuition is as follows. When deciding whether to initiate collusion in this way, a supplier faces a tradeoff that is similar to the tradeoff driving the decision about whether to maintain the market allocation: initiating collusion increases future profits but involves a short-term sacrifice. The long-term stakes are the same for the two decisions, and correspond to the difference between collusive and competitive profits. The short-term stakes potentially differ. Initiating collusion by bidding the reserve reduces the supplier's expected profit from  $\pi^c(c)$  to  $\pi^n(c)$ , as the supplier then obtains a profit (equal to  $r - c$ ) only when the other supplier's cost exceeds the reserve. By contrast, deviating from collusion would increase the supplier's profit from  $\pi^n(c)$  to the deviation profit  $\pi_a^d(c)$ , which depends on the auction format,  $a \in \{SPA, FPA\}$ . Under second-price auctions, the two short-term sacrifices actually coincide, as the designated supplier bids its cost anyway (under both collusion and competition): as we have seen,  $\pi_{SPA}^d(c) = \pi^c(c)$ . It follows that initiating collusion is profitable exactly when it is sustainable. By contrast, under first-price auctions, the designated supplier bids close to the reserve in case of a market allocation, and more aggressively in case of competition; as a result, the short-term sacrifice is higher for sustainability than for initiation: for sustainability, it is based on the monopoly profit that can be stolen away from the designated bidder, whereas for initiation, it is based instead on the competitive profit; hence,  $\pi_{FPA}^d(c) = \pi^m(c) > \pi^c(c)$ . It follows that collusion is easier to initiate than to sustain.

Proposition 2 implies that, even when accounting for the need to initiate collusion, the key conditions determining the feasibility of collusion remain those driving sustainability:

**Corollary 1.** *When considering both initiation and sustainability, the most stringent feasibility conditions are those for sustainability. Hence, from Proposition 1, collusion is easier to sustain and initiate under staggered rather than synchronized purchasing, and with second-price rather than first-price auctions.*

Corollary 1 shows that, contrary to the concern mentioned above in the context of the AT&T–Time Warner merger, collusion may not be more difficult to initiate than to sustain, and may be facilitated, rather than impeded, by staggered contracts.

*Remark: on the initiator's cost.* The discussion of the initiation of collusion compares the conditions for sustainability and for the profitability of initiating collusion for any given cost realization  $c$ . Both conditions become tighter for lower cost realizations. However,

sustainability requires the associated condition to hold for *every* cost realization that lies below the reserve; hence, it must hold for the lowest cost realization. By contrast, in principle, collusion could be initiated for some cost realizations even if it could not be initiated for the lowest cost realization. Taking this into consideration tends to further downplay the difficulty of initiating collusion. Put otherwise, our proposition for the baseline setting is quite strong, as it shows that, whenever collusion is sustainable, it can be initiated as soon as at least one supplier has a cost below the reserve, that is, *as soon as the tender would be successful under competition*.

### 3.4 Sales auctions

For the sake of exposition, we have presented our analysis in the context of procurement auctions, but it applies equally to the case of sales auctions: it suffices to swap the roles of buyers and suppliers.

Consider for instance an environment where a supplier (or equivalently, two suppliers) repeatedly sells two scarce resources, each one constituting a “market,” at discrete points in time over an infinite horizon. In the spirit of the above setting, assume that: (i) there are two buyers, who privately and independently draw their values (which could be either market specific, or the same in both markets) from distribution  $G$  over  $[\underline{v}, \bar{v}]$ ; and (ii) the supplier has a reservation value of  $r \in (\underline{v}, \bar{v})$ , ensuring that there are gains from trade.

In this setting, our analysis carries through by defining a buyer’s monopsony payoff per market, given its value  $v$ , as

$$\pi^m(v) \equiv \min \{0, v - r\},$$

and a buyer’s competitive payoff per market, given its value  $v$ , as

$$\pi^c(v) \equiv \mathbb{E}_{\tilde{v}} [\max \{0, v - \max \{\tilde{v}, r\}\}].$$

## 4 Buyer defensive measures

Market allocation schemes can be difficult to detect, especially given incentives for colluding agents to disguise their conduct. However, suspicions may be aroused by certain bidding patterns, such as bids that are consistently close to the reserve, or a supplier withdrawing supply from a market. In this section, we consider various tools that a buyer can use to “fight back” when suspecting collusion. Buyers may differ in their ability to use these tools based, for example, on their sophistication, commitment ability, and purchase volumes.

In Section 4.1, we show that imposing a sufficiently aggressive reserve can inhibit collusion. As noted in Cassady (1967, p. 166), the use by auctioneers of reserves to address collusion is standard practice.<sup>22</sup> This plays out in our setup by giving a buyer an incentive to adjust its reserve downwards. This result lends support to advice given to procurement designers that “When collusion among suppliers is suspected, the reserve price should be set at a lower value than in the absence of collusion” (Albano et al., 2006b, p. 282).

In Section 4.2, we consider contract duration and show that collusion becomes more difficult as the contract length grows longer. This is consistent with results in the theoretical literature that a lower frequency of interaction makes collusion more difficult (see e.g., Tirole, 1988; Ivaldi et al., 2003) and supported by experimental findings (e.g., Friedman and Oprea, 2012; Bigoni et al., 2018). The analysis also extends our insight on purchasing patterns: collusion is easiest when auctions are staggered as evenly as possible, and most difficult when they are exactly synchronous.

In Section 4.3, we analyze the effects of a buyer withholding information on placed bids. We show that collusion becomes more difficult to initiate and sustain than when bids are not observable.

## 4.1 Reserves

We first note below that an aggressive reserve discourages collusion.<sup>23</sup> We then study the buyers’ optimal choices of reserves.

### More aggressive reserves discourage collusion

The conditions for sustaining a market allocation hinge on the expected profitability of future collusion and on the gain from deviating for a very efficient supplier (i.e., with cost  $\underline{c}$ ). Obviously, decreasing the reserve reduces suppliers’ profits—under both collusion and competition. Critically, however, when the reserve is already low, the *expected* profitability of collusion decreases at a faster rate than the deviation gain for an efficient supplier; this is because the reserve is likely to prevent future trade, whereas an efficient supplier always trades. It follows that, for sufficiently low reserves, the threshold discount factors

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<sup>22</sup>Graham et al. (1996) consider a particular form of dynamic reserve price adjustment where the reserve adjusts based on observed bids and show that it is optimal in certain settings. According to Graham et al. (1996, fn. 3), dynamic reserve price adjustment is “a pervasive phenomenon at auctions for horses and at bankruptcy liquidation auctions.”

<sup>23</sup>The potential for buyer power to limit the ability of firms to sustain collusion, particularly tacit collusion, is recognized in the literature (Loertscher and Marx, 2019a) and by competition authorities (see, e.g., Section 8 on “Powerful Buyers” in the U.S. DOJ and FTC’s Horizontal Merger Guidelines).

are decreasing in the reserve, and a sufficiently aggressive reserve can prevent a market allocation from being sustainable.

**Proposition 3.** *Assuming that  $g(\underline{c}) > 0$ ,<sup>24</sup> there exists  $\bar{r} \in (\underline{c}, \bar{c}]$  such that in the range  $r \in [\underline{c}, \bar{r}]$ , the critical discount factors for synchronized and staggered purchasing are both decreasing in the reserve; furthermore, these thresholds tend to 1 as  $r$  tends to  $\underline{c}$ .*

*Proof.* See Appendix A.2.<sup>25</sup>

Proposition 3 implies that a buyer with concerns that its suppliers might engage in a market allocation may have an incentive to use a more aggressive reserve, regardless of whether purchasing is synchronized or staggered. This suggests another type of inefficiency associated with collusive conduct—the threat of collusive conduct among suppliers may induce buyers to use procurement practices that are less efficient.

As an illustration, letting  $\delta_a^s(r)$  denote the threshold discount factor for the sustainability (and initiation) of collusion for timing  $s$  and auction format  $a$ , Figure 2 displays the threshold discount factors for  $r \in [\underline{c}, \bar{c}]$  when costs are uniformly distributed over  $[0, 1]$ .<sup>26</sup>

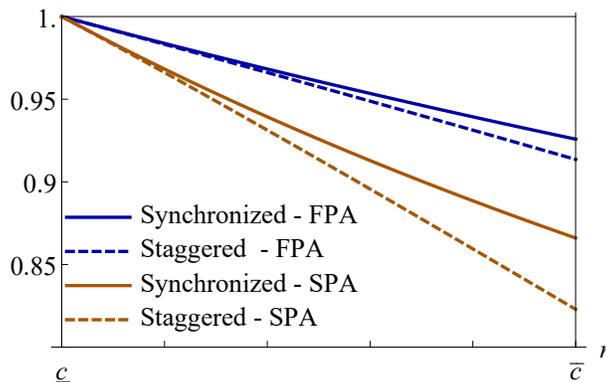


Figure 2: Threshold discount factors for synchronized and staggered purchasing and first-price and second-price auctions as a function of the reserve  $r$  when costs are uniformly distributed over  $[0, 1]$ .

Consistent with Proposition 3, the functions are decreasing in  $r$  and go to 1 and  $r$  goes to  $\underline{c}$ . The figure also illustrates the insights of Proposition 1: for any given reserve and

<sup>24</sup>The assumption that  $g(\underline{c}) > 0$  allows us to sign the derivatives of the critical discount factors with respect to the reserve as  $r$  approaches  $\underline{c}$ . Because we assume that  $g(c)$  is positive for  $c \in (\underline{c}, \bar{c})$ , there is some finite integer  $k$  such that the  $k$ -th derivative of  $g(\underline{c})$  is positive, and so the assumption that  $g(\underline{c}) > 0$  can be dropped at the cost of an extended proof.

<sup>25</sup>The critical discount factors need not be decreasing in  $r$  over the entire range  $r \in [\underline{c}, \bar{c}]$ . For example, the Beta distribution with parameters  $(1/2, 1/2)$  generates a critical discount factor that is increasing for  $r$  close to  $\bar{c}$ .

<sup>26</sup>In that case,  $\delta_{FPA}^{Sync}(r) = \sqrt{6}/\sqrt{6+r}$ ,  $\delta_{FPA}^{Stag}(r) = (\sqrt{36-6r+r^2}-r)/(6-r)$ ,  $\delta_{SPA}^{Sync}(r) = \sqrt{3}/\sqrt{3+r}$ , and  $\delta_{SPA}^{Stag}(r) = (\sqrt{9-3r+r^2}-r)/(3-r)$ .

purchase timing, the threshold discount factor is greater for a first-price auction than for a second-price auction; and for any given reserve and auction type, the threshold discount factor is greater for synchronized purchasing than for staggered purchasing.

In addition, Figure 2 shows that for the uniform distribution, the threshold discount factor is lower for second-price auctions and synchronized purchasing than for first-price auctions and staggered purchasing:

$$\delta_{SPA}^{Stag}(r) < \delta_{SPA}^{Sync}(r) < \delta_{FPA}^{Stag}(r) < \delta_{FPA}^{Sync}(r).$$

Hence, starting from the most collusion-prone scenario of a second-price auction with staggered purchasing, a change in the auction format has a greater impact on deterrence than a change in the timing of purchasing: buyers would get greater deterrence by changing the auction format to a first-price auction rather than by switching to synchronized purchasing.

### Optimal reserve under collusion

In order to study the buyers' best use of reserves as a defensive measure, it is useful to first characterize the optimal reserve when suppliers do collude and compare it with the optimal reserve under competition. When the suppliers bid competitively and face a reserve  $r \in [\underline{c}, \bar{c}]$ , a buyer's payoff is

$$U^{Comp}(v, r) = 2(v - r)[1 - G(r)]G(r) + 2 \int_{\underline{c}}^r (v - c)G(c)g(c)dc, \quad (16)$$

where  $2[1 - G(r)]G(r)$  is the probability that exactly one cost draw is above and one below the reserve, and  $2G(c)g(c)$  is the probability density function of the higher cost (with cumulative distribution function  $G^2(c)$ ). Under collusive bidding, the buyer's expected payoff is instead

$$U^{Coll}(v, r) = (v - r)\hat{G}(r), \quad (17)$$

where  $\hat{G}(c) \equiv 1 - [1 - G(c)]^2$  is the cumulative distribution function of the lower cost. In what follows, we let  $\hat{g}(c)$  denote the associated density.

The buyer is strictly better off with competitive rather than collusive bidding: in both cases, trade takes place under the same condition (namely, whenever at least one supplier's cost lies below the reserve); however, when it does take place, the buyer always pays the reserve in case of collusion, whereas it benefits from a lower price under competitive bidding. Integrating (16) by parts, the "competition benefit" can be expressed as:

$$B(r) \equiv U^{Comp}(v, r) - U^{Coll}(v, r) = \int_{\underline{c}}^r G^2(c)dc. \quad (18)$$

Let  $r^{Comp}(v)$  denote the optimal reserve in the absence of collusion and  $r^{Coll}(v)$  denote the optimal reserve in the presence of collusion. The fact that the buyer always pays the reserve when it faces collusion suggests that it should set a more aggressive reserve in that case. A simple revealed preference argument confirms that intuition, by establishing that the competition benefit  $B(r)$  must be greater for  $r^{Comp}(v)$  than for  $r^{Coll}(v)$  (see Appendix A.3 for a formal derivation). As  $B(r)$  increases with  $r$ , it follows that  $r^{Comp}(v) \geq r^{Coll}(v)$ . Inspection of the payoffs under collusion and competition shows that this ranking is moreover strict:<sup>27</sup>

**Proposition 4.** *The optimal reserve is strictly more aggressive when facing collusion:  $r^{Coll}(v) < r^{Comp}(v)$ .*

*Proof.* See Appendix A.3.

Note that the buyer's benefit from competition,  $B(r)$ , is equal to the designated supplier's benefit from collusion,  $\bar{\pi}^m(r) - \bar{\pi}^c(r)$ . This is because the designated supplier benefits from collusion if and only if both costs are below the reserve, which are precisely the instances in which the buyer benefits from competitive bidding. In other words, from the perspective of the designated supplier and the buyer, the market allocation scheme is merely a transfer. However, because the market allocation scheme is inefficient, it follows that the reduction in social surplus that it causes must be borne, entirely, by the non-designated supplier.<sup>28</sup>

To complete the characterization of the optimal thresholds, note that

$$\frac{\partial U^{Comp}}{\partial r}(v, r) = g(r)[v - \Gamma(r)] \quad \text{and} \quad \frac{\partial U^{Coll}}{\partial r}(v, r) = \hat{g}(r)[v - \hat{\Gamma}(r)],$$

where

$$\Gamma(c) \equiv c + \frac{G(c)}{g(c)} \quad \text{and} \quad \hat{\Gamma}(c) \equiv c + \frac{\hat{G}(c)}{\hat{g}(c)} = c + \frac{G(c)}{g(c)} \frac{2 - G(c)}{1 - G(c)}$$

---

<sup>27</sup>It is interesting to note the similarity and subtle difference relative to Blume and Heidhues (2004). As mentioned previously, their analysis implies that in a one-shot, second-price auction, any reserve below  $\bar{c}$  eliminates collusive Bayes Nash equilibria, whereas with  $r \geq \bar{c}$ , there are a continuum of collusive Bayes Nash equilibria. In our setting, the optimal reserve in the face of collusion is more aggressive than with competitive bidding.

<sup>28</sup>Formally, the cost of collusion for the non-designated supplier is  $\bar{\pi}^c(r) - \bar{\pi}^n(r) = \int_{\underline{c}}^r [G(r) - G(c)]G(c)dc$ . The change in social surplus is equal to the change in the cost of production, which is equal to  $\int_{\underline{c}}^r cdG(c) + [1 - G(r)] \int_{\underline{c}}^r cdG(c)$  under collusion, and to  $\int_{\underline{c}}^r cd\hat{G}(c)$  under competition. Integrating by parts and simplifying confirms that the difference in costs is indeed equal to  $\int_{\underline{c}}^r [G(r) - G(c)]G(c)dc$ .

are the *virtual costs* associated with the distributions  $G$  and  $\hat{G}$ . From now on, we assume that these virtual costs are strictly increasing. This ensures that a buyer's payoffs are strictly quasiconcave in  $r$  for any  $v$ ; the optimal reserves are then given by

$$r^{Comp}(v) = \min \{ \Gamma^{-1}(v), \bar{c} \} \quad \text{and} \quad r^{Coll}(v) = \hat{\Gamma}^{-1}(v) < \bar{c},$$

where the last inequality follows from  $\hat{\Gamma}(\bar{c}) = \infty$ .

Under competition, the optimal reserve,  $r^{Comp}(v)$ , corresponds to the monopsony price that a buyer would charge, given its valuation  $v$ , if it were to face a single supplier with cost distribution  $G$ .<sup>29</sup> Perhaps more surprisingly,  $r^{Coll}(v)$  is the reserve that would be optimal if the two suppliers had merged (Loertscher and Marx, 2019b), a situation equivalent to perfect collusion. The market allocation scheme does not achieve perfect collusion because production does not necessarily occur at the lowest cost; however, this is immaterial for the buyer: what matters is whether production occurs, in which case the buyer pays the reserve, and it occurs in exactly the same instances as under perfect collusion, namely, when at least one supplier has a cost below the reserve. It follows that the optimal reserve is the monopsony price that a buyer would charge when facing a single supplier with the *enhanced* cost distribution  $\hat{G}$ , which corresponds to the lower of two draws. This distribution has a lower reverse hazard rate than the original distribution  $G$ , which makes the supply less elastic and leads to a more aggressive reserve (because  $\hat{\Gamma}^{-1}(v) < \Gamma^{-1}(v)$ ).

*Remark: on the nature of collusion.* The above market allocation scheme enables the non-designated supplier to step in when trade would otherwise not occur. The alternative rotation scheme, requiring the non-designated supplier to withdraw regardless of the realized costs, amounts instead to a reduction in the number of suppliers. Because the optimal reserve is independent of the number of bidders, buyers' optimal reserves are then the same as under competition. Hence, the optimal reserve depends not only on whether suppliers collude, but also on the nature of the collusive scheme; it may, moreover, decrease as collusion becomes more efficient.

### **To accommodate or to deter collusion?**

We now study the question of whether buyers want to accommodate or deter collusion. For this purpose, we assume here that the reserves are the same across the two markets; in Section 5.5, we extend the analysis to allow asymmetric reserves. We also assume that, for  $s \in \{Sync, Stag\}$  and  $a \in \{SPA, FPA\}$ , there is a well-defined reserve threshold,  $r_a^s(\delta)$ , such that collusion is sustainable if and only if the reserve exceeds that threshold.

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<sup>29</sup>As is well-known, the optimal reserve does not depend on the number of suppliers; hence, it maximizes the monopsony profit  $(v - r)G(r)$ .

Obviously, collusion is not an issue when the actual discount factor  $\delta$  is small enough (e.g.,  $\delta < \delta_a^s(\bar{c})$ , implying  $r_a^s(\delta) = \bar{c}$ ). For the sake of exposition, we assume throughout the remainder of this discussion that collusion *is* a concern:

$$r_a^s(\delta) < \bar{c}.$$

Preventing collusion then requires sacrificing some trade, which may not be in the best interest of the buyers. First, this may be too costly. This is the case, for example, when  $r_a^s(\delta)$  is close to  $\underline{c}$  and the value  $v$  is large. In this case, preventing collusion comes at the cost of canceling the procurement with a probability close to 1, which is disastrous when trade is highly valuable. Second, as we analyze next, there may be better means for buyers to address collusion among the suppliers, for example, by setting optimal reserves in the face of collusion.

To fix ideas, consider a comparative statics exercise in which all parameters are fixed except for  $v$ . To assume away any coordination issue, we first consider the case in which *a single buyer* is running both procurements (e.g., for different products or geographic areas); we discuss potential coordination problems among multiple buyers later on. From the above analysis, the optimal reserves under competition and collusion satisfy  $r^{Coll}(v) < r^{Comp}(v) < v$ , and they increase with  $v$ . Hence, for  $v$  sufficiently small, namely:

$$v < \underline{v}(\delta) \equiv \Gamma(r_a^s(\delta)), \tag{19}$$

we have  $r^{Comp}(v) < r_a^s(\delta)$ . In this case, it is optimal for the buyer to set the optimal reserve under competitive bidding. Using terminology from the literature on entry deterrence, collusion can be said to be *blocked* because the optimal reserve absent collusion,  $r^{Comp}(v)$ , is sufficiently low that it deters collusion.

For higher values of  $v$ , namely:

$$\underline{v}(\delta) \leq v \leq \bar{v}(\delta) \equiv \hat{\Gamma}(r_a^s(\delta)), \tag{20}$$

we have  $r^{Coll}(v) \leq r_a^s(\delta) \leq r^{Comp}(v)$ , in which case the quasiconcavity of the buyer's payoff ensures that the buyer is best off with the reserve  $r_a^s(\delta)$ . In the range yielding competition (i.e., for  $r \leq r_a^s(\delta)$ ), the buyer's payoff increases in  $r$ , whereas in the range that gives rise to collusion (i.e., for  $r \geq r_a^s(\delta)$ ), the buyer's payoff decreases in  $r$ . Because the buyer is better off with competition, it is therefore optimal for the buyer to *deter* collusion by setting the reserve slightly below  $r_a^s(\delta)$ .<sup>30</sup>

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<sup>30</sup>When the reserve is set to the deterrence threshold, collusion is “barely” sustainable; to prevent it for sure, buyers should thus set a reserve below (but arbitrarily close to) the threshold. As always with deterrence, maintaining a threat begs the question of credibility. Although we do not model this explicitly,

Finally, for  $v > \bar{v}(\delta)$ , we have  $r_a^s(\delta) < r^{Coll}(v)$ , in which case the buyer faces a non-trivial tradeoff between *accommodating* collusion by choosing  $r^{Coll}(v)$  or deterring it by choosing a more aggressive reserve slightly below  $r_a^s(\delta)$ . The quasiconcavity of the payoff under collusion ensures that the buyer is better off deterring collusion if  $r_a^s(\delta)$  is close to  $r^{Coll}(v)$  because then the loss arising from setting the aggressive reserve  $r_a^s(\delta)$  is second-order relative to the (discrete) benefit of inducing competitive bidding. Conversely, as the distance between  $r_a^s(\delta)$  and  $r^{Coll}(v)$  increases, deterring collusion eventually becomes too costly. The buyer will then swallow the bitter pill of collusion and set the optimal reserve in the face of collusion,<sup>31</sup> as shown in the following proposition—recall that here we assume that the virtual costs  $\Gamma$  and  $\hat{\Gamma}$  are strictly increasing and that there is a well-defined reserve threshold for the sustainability of collusion that is less than  $\bar{c}$ :

**Proposition 5.** *Letting  $\underline{v}(\delta)$  and  $\bar{v}(\delta)$  denote the thresholds defined by (19) and (20), there exists  $\hat{v} > \bar{v}(\delta)$  such that: (i) when  $v < \underline{v}(\delta)$ , collusion is blockaded; (ii) when  $\underline{v}(\delta) \leq v < \hat{v}$ , it is optimal for the buyer to deter collusion; and (iii) when  $v \geq \hat{v}$ , it is optimal for the buyer to accommodate collusion.*

*Proof.* See Appendix A.4.

Using Proposition 5 and our monotonicity assumptions, it follows that the optimal symmetric reserve for the buyers is given by:

$$\begin{cases} r^{Comp}(v) & \text{if } \delta \leq \underline{\delta}_a^s(v), \\ r_a^s(\delta) & \text{if } \delta \in (\underline{\delta}_a^s(v), \bar{\delta}_a^s(v)], \\ r^{Coll}(v) & \text{otherwise,} \end{cases}$$

where  $\underline{\delta}_a^s(v)$  is the largest discount factor such that there is competition at a reserve of  $r^{Comp}(v)$  and  $\bar{\delta}_a^s(v)$  is the largest discount factor such that deterrence is optimal for the buyers.

It then follows that the buyer's optimal reserve is not monotone in the discount factor. As the discount factor increases from zero, the optimal reserve is initially constant at  $r^{Comp}(v)$ . Then, once the discount factor is sufficiently large that collusion is not blockaded, the optimal reserve shifts to  $r_a^s(\delta)$ , which is a decreasing function of the discount

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the repeated interaction setting under scrutiny may help the buyer to maintain a credible threat.

<sup>31</sup>In principle, the buyer could also contemplate contingent strategies, such as setting the optimal reserve  $r^{Comp}(v)$  in case of competitive bidding and reverting to a “grim trigger” strategy of setting  $r_a^s(\delta)$  forever if the suppliers are caught (or believed to be) colluding. If the suppliers were to consider this strategy as credible, they would refrain from colluding, which would yield the first-best outcome for the buyer.

factor. As the discount factor increases further, assuming that collusion is a concern, eventually  $U^{Comp}(v, r_a^s(\delta)) = U^{Coll}(v, r^{Coll}(v))$ , at which point deterring collusion becomes too costly, and the optimal reserve jumps up to the optimal collusive reserve. Figure 3 illustrates this for the case of costs that are uniformly distributed over  $[0, 1]$  and a buyer with value  $v = 1$ .<sup>32</sup>

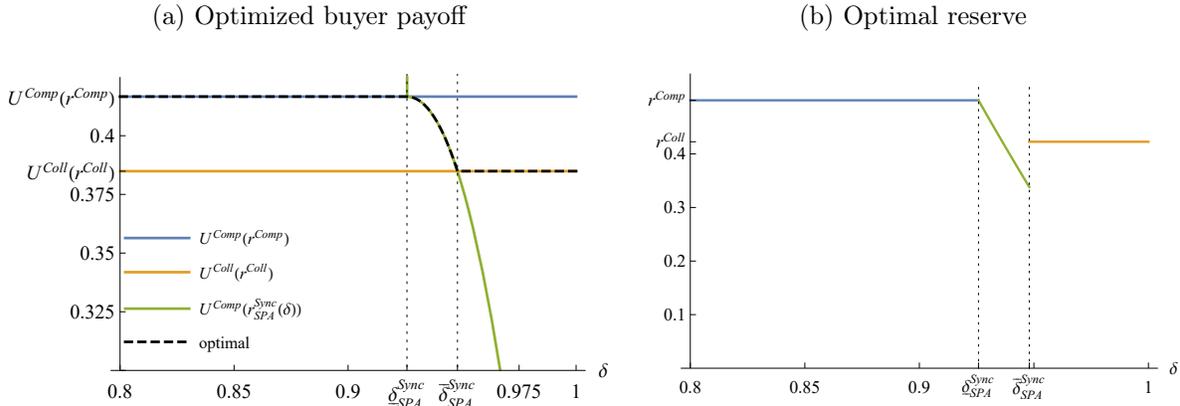


Figure 3: Illustration of buyer's expected payoff under the optimal reserve and the optimal reserve itself, as functions of the discount factor, under synchronized purchasing and second-price auctions. Assumes that costs are uniformly distributed over  $[0, 1]$  and  $v = 1$ .

As shown in Figure 3(a), for sufficiently low values of  $\delta$ , the buyer can achieve a payoff of  $U^{Comp}(v, r^{Comp}(v))$ ; for intermediate values of  $\delta$ , the buyer's maximized payoff is  $U^{Comp}(v, r_a^s(\delta))$ ; and for sufficiently high values of  $\delta$ , the buyer's maximized payoff is  $U^{Coll}(v, r^{Coll}(v))$ . Figure 3(b) shows the optimal reserve as a function of  $\delta$ .

Although this analysis assumes that the same reserve is used in both markets, deterrence strategies that involve setting different reserves across the two markets might be less costly for the buyer, depending on the response of the suppliers to those differential reserves.<sup>33</sup> Of course, with asymmetric reserves, additional collusive strategies beyond a market allocation potentially become relevant. For example, collusion might be easier

<sup>32</sup>In this case, we have  $r_{SPA}^{Sync}(\delta) = 3(1 - \delta^2)/\delta^2$ .

<sup>33</sup>For example, focusing on suppliers' use of a market allocation, if reserves are asymmetric, then, defining the long-term stake as

$$L^{Sync}(\delta, r_i, r_j) \equiv \frac{\delta^2}{1 - \delta^2} (\bar{\pi}^m(r_i) + \bar{\pi}^n(r_j) - \bar{\pi}^c(r_i) - \bar{\pi}^c(r_j)),$$

a market allocation is sustainable if and only if

$$L^{Sync}(\delta, r_1, r_2) \geq S_a(r_2) \quad \text{and} \quad L^{Sync}(\delta, r_2, r_1) \geq S_a(r_1),$$

where the first condition is the relevant one when  $r_2 > r_1$ . This implies that, starting from optimal symmetric deterrence reserves, a market allocation remains deterred if  $r_2$  is increased, holding  $r_1$  constant:

to sustain if the suppliers alternate markets rather than having one supplier always “assigned” to the market with the lower reserve. We address these issues in Section 5.5 and show that the general characterization of the buyer’s optimal deterrence strategy remains the same: for sufficiently low discount factors, the buyer uses the optimal competitive reserve; for sufficiently high discount factors, the buyer uses the optimal collusive reserve; and for intermediate discount factors, the buyer sets (asymmetric) reserves so as to deter collusion. We also discuss the possibility of coordination failure when, instead of having one buyer that operates in both markets, there are two separate buyers.

### Secret reserves

Li and Perrigne (2003, p. 189) note that “The theoretic auction literature is still unclear on the rationale for using a random reserve price.”<sup>34</sup> However, in our setup, opting for a *secret*, random reserve does create challenges for initiating and sustaining a market allocation.

First, secret, random reserves prevent suppliers from signaling their wish to initiate collusion through a bid equal to the reserve, because they do not know what that reserve is. That said, if the reserve is drawn from a distribution with upper bound of the support  $\bar{r} < \bar{c}$ , there remains the possibility of signaling initiation with a bid of  $\bar{r}$ . Second, secret, random reserves inhibit the ability of suppliers to maintain a market allocation while still having positive expected payoffs in their non-designated markets. To see this, note that with a secret, random reserve drawn from a distribution with upper bound of support  $\bar{r}$ , the only way for the non-designated supplier to ensure that it does not provide meaningful competition for the designated supplier is to bid  $\bar{r}$  or above. But in this case, unless it bids exactly  $\bar{r}$  and there is an atom in the distribution at that point, it wins with probability zero, and so it has an expected payoff of zero in its non-designated market. As these points suggest, secret, random reserves create challenges for colluding suppliers, and more so if those reserves are drawn from a distribution whose upper support is  $\bar{r} = \bar{c}$ .

Further, the possibility of using secret, random reserves suggests that when  $r_a^s(\delta) < r^{Comp}(v)$ , a buyer could deter collusion, and obtain a higher expected payoff than with a fixed reserve of  $r_a^s(\delta)$ , by using a secret reserve that is randomly chosen from  $[r_a^s(\delta), r^{Comp}]$ . By increasing the profitability of deterrence, such a strategy could expand the range of

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this increases the short-term stake  $S_a(r_2)$  as well as the cost of collusion for the non-designated supplier, which is equal to  $\bar{\pi}^c(r_2) - \bar{\pi}^n(r_2)$ . It follows that the deterrence of a market allocation is optimally achieved using asymmetric reserves.

<sup>34</sup>Li and Perrigne (2003) estimate that the French forest service had lower profits as a result of using secret, random reserve prices at timber auctions; however, they assume noncooperative bidding. Given evidence of collusion at U.S. timber auctions (see, e.g., Baldwin et al., 1997; Athey and Levin, 2001), the secret, random reserve may have provided benefits in terms of deterring collusion not captured in an analysis based on noncooperative bidding.

values for which a buyer prefers to deter rather than accommodate collusion.

## 4.2 Contract duration

In this section, we consider the effects of contract length. Specifically, we assume that contracts last for  $T$  periods and the first buyer organizes tenders at  $t \in \{1, T+1, 2T+1, \dots\}$ , whereas the second buyer organizes them at  $t \in \{\tau, T+\tau, 2T+\tau, \dots\}$ , where  $\tau \in \{1, \dots, T\}$ . The case of synchronous purchasing corresponds to  $\tau = 1$ , whereas the case of “perfectly” staggered purchasing corresponds to  $\tau = T/2 + 1$  if  $T$  is an even number and to  $\tau \in \{(T+1)/2, (T+3)/2\}$  otherwise (both options are equivalent and constitute the best approximations of perfectly staggered purchases). For the sake of exposition, we follow the interpretation described in Section 2’s remark on the timing of suppliers’ costs, according to which the cost of fulfilling the contractual obligations in any given period  $t$ ,  $c^t$ , is observed by the supplier at the beginning of that period. Because the buyer and the suppliers have the same information about future costs, suppliers derive their profits from the informational advantage associated with the first period of the contract;<sup>35</sup> the relevant expressions for  $\pi^c(c)$ ,  $\pi^m(c)$  and  $\pi^n(c)$ , as well as those of  $\bar{\pi}^c$ ,  $\bar{\pi}^m$  and  $\bar{\pi}^n$ , thus remain unchanged as  $T$  varies, and are the same as in the baseline model.<sup>36</sup> We assume further that the market is at risk, that is, that collusion is profitable:  $\bar{\pi}^m + \bar{\pi}^n > 2\bar{\pi}^c$ .

The short-term stake remains given by (8), but the long-term stake is affected both by the contract length  $T$  and the timing of purchases, characterized by  $\tau$ . We show in Appendix A.5 that it is now given by:

$$L(\delta; T, \tau) \equiv \frac{\delta^{\alpha(T, \tau)}}{1 - \delta^T} (\bar{\pi}^m - \bar{\pi}^c) - \frac{\delta^T}{1 - \delta^T} (\bar{\pi}^c - \bar{\pi}^n), \quad (21)$$

where  $\alpha(T, \tau) \equiv \max\{\tau - 1, T + 1 - \tau\}$  denotes the maximal lag between two tenders. This long-term stake decreases in  $\alpha$ : the sustainability and initiation conditions become more stringent as  $\alpha(T, \tau)$  increases. It follows that the scope for collusion is minimized when the buyers perfectly synchronize their purchases ( $\tau = 1$ ) and maximized when procurements are perfectly staggered ( $\tau$  as close as possible to  $T/2 + 1$ ). However, for both synchronous and staggered purchasing, because the long-term stake is decreasing in

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<sup>35</sup> The expected cost of fulfilling the contractual obligations of a  $T$ -period contract awarded in period  $t$  is thus  $c^t + (\delta + \dots + \delta^{T-1})\mathbb{E}_c[c]$ , and the buyer can set a reserve equal to  $\mathbb{E}_c[c]$  for the future periods; the total reserve can thus be expressed as  $R \equiv r + (\delta + \dots + \delta^{T-1})\mathbb{E}_c[c]$ , where  $r$  denotes, as before, the relevant reserve for the first period of the contract.

<sup>36</sup>The alternative interpretation, according to which the buyer purchases what it needs to support its production for the duration of the contract, would amount to increase the affected volumes  $T$  times. The short-term and long-term stakes would therefore all be multiplied by  $T$ , which could be factored out, and the analysis would remain unchanged.

$T$ , collusion becomes more difficult to sustain and initiate as  $T$  increases. This leads to:

**Proposition 6.** *For any given contract duration, collusion is most difficult to sustain with synchronous purchasing and easiest to sustain with staggered purchasing. Furthermore, for both synchronized and maximally staggered purchasing, collusion becomes more difficult to sustain and initiate as contracts become longer.*

*Proof.* See Appendix A.5.

The intuition builds on our previous insights. Maximizing the gap between *successive* tenders helps to fight collusion because it delays the punishment phase. This requires increasing contract length and spacing out successive tenders as much as possible, which is best achieved with synchronous purchases.

### 4.3 Withholding information

In this section, we show that a buyer may be able to deter collusion by withholding information about the submitted bids. We consider two cases. In the first case, the buyer withholds information on all bids, making the award price unobservable to the losing bidder (except in the case of a second-price auction, where a losing bidder whose bid was below the reserve can infer the award price). This case is relevant for private procurement, as private buyers typically have no transparency obligations towards their suppliers or third parties. In the second case, the buyer withholds information on bids, but the award price is observable by the losing bidder. This case is common for public procurement, where transparency and accountability obligations often require that the award price be made public.

Withholding information about the bids does not prevent the suppliers from using the collusive mechanism as such: they can still allocate markets, and the non-designated supplier can still bid the reserve, as before. It follows that the long-term stakes—conditional on being detected, see below—are not affected by the information regime; we can thus focus on the short-term stakes.

For a second-price auction with observable bids, the short-term stake is  $\underline{S} \equiv S_{SPA}$ ; for both sustainability and initiation. When bids are not observable, regardless of whether the award price is observable, a deviation by the non-designated supplier or an attempt to signal initiation is only observed when the market’s designated supplier has a cost below the reserve, which has probability  $G(r)$ . Thus, the relevant short-term stake is the gain from the deviation, *conditional on being detected*, which is equal to  $\underline{S}/G(r)$ .<sup>37</sup>

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<sup>37</sup>The deviation is detected when the designated supplier’s cost lies below the reserve, and brings no benefit otherwise (as the non-designated bidder obtains the reserve anyway); hence, the conditional short-term stake  $S^c$  satisfies  $\underline{S} = G(r) S^c + [1 - G(r)] \times 0$ , or  $S^c = \underline{S}/G(r)$ .

For a first-price auction, when bids are not observable the short-term stakes are  $\underline{S}$  for initiation and  $\bar{S} \equiv S_{FPA} > \underline{S}$  for sustainability. Suppose now that individual bids are not observable. If the award price remains public, deviations are still always detected, and so sustainability is unaffected. By contrast, attempts to signal initiation are only observed when the designated supplier has a cost above the reserve, which has probability  $1 - G(r)$ . It follows that the relevant short-term stake for initiation is the profit lost, *conditional on the signal being observed*, which is equal to  $\underline{S}/[1 - G(r)]$ . If instead the award price is not observable either, then: (i) deviations are only observed when the market’s designated supplier has a cost below the reserve, which has probability  $G(r)$ , so the relevant short-term stake for sustainability becomes  $\bar{S}/G(r)$ ; and (ii) attempts to signal initiation are never observed.

Table 1 presents the relevant short-term stakes for the two auction formats, as a function of the information regime.

Table 1: Short-term stakes for different observability regimes

	Public bids		Public award price		No public information	
	Sustain	Initiate	Sustain	Initiate	Sustain	Initiate
FPA	$\bar{S}$	$\underline{S}$	$\bar{S}$	$\underline{S}/[1 - G(r)]$	$\bar{S}/G(r)$	“∞”
SPA		$\underline{S}$		$\underline{S}/G(r)$		$\underline{S}/G(r)$

Summarizing the above analysis, we have:

**Proposition 7.** *Withholding information on bids and on the award price precludes initiation under second-price auctions. Withholding information on individual bids (leaving the award price public): (i) for second-price auctions, impedes both initiation and sustainability whenever  $r < \bar{c}$ ; and (ii) for first-price auctions, makes initiation more difficult and, for sufficiently high reserves, improves deterrence.*

Withholding information can thus contribute to deter collusion. In the case of second-price auctions, withholding information in individual bids suffices to achieve the desired effect, even if the award price remains public. By contrast, in the case of first-price auctions, withholding information about the award price can be useful as well. Furthermore, whenever withholding information improves deterrence for this auction format, it does so primarily not through reduced sustainability, but rather by creating challenges for initiation: this is particularly clear when all information is withheld, as this somewhat impedes

sustainability but makes initiation very difficult if not impossible, but is true as well when the award price is not made public, in which case withholding information on individual bids only affects initiation, but indeed improves deterrence when the reserves are high (as initiation then becomes more challenging than sustainability).

Interestingly, if only the award price remains public, then, for low reserves it is actually more difficult to sustain and initiate collusion with second-price auctions than it is to sustain and, a fortiori, to initiate collusion with first-price auctions. This contrasts with the usual thinking that first-price auctions are less susceptible to collusion than second-price auctions:

**Corollary 2.** *When only the award price remains public and the reserve is low enough, collusion is easier with second-price auctions rather than first-price auctions.*

This feature is illustrated by Figure 4 which shows that, when costs are uniformly distributed over  $[0, 1]$ , collusion is easier with first-price auctions whenever the reserve is less than half of the maximal value of the suppliers' cost.<sup>38</sup>

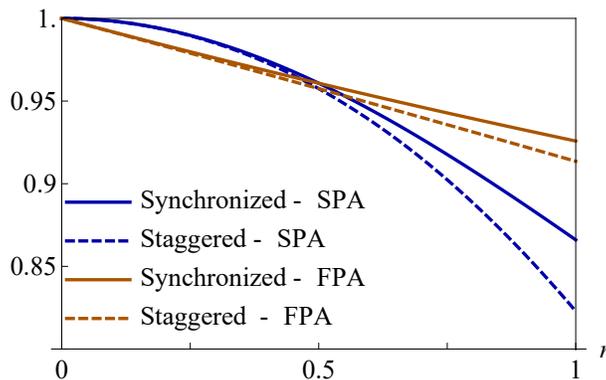


Figure 4: Threshold discount factors when the buyer can withhold information on bids, but the award price is observable. Assumes that costs are uniformly distributed over  $[0, 1]$ .

## 5 Extensions

In this section we explore several extensions. In Section 5.1, we show that our analysis can be applied to classic product market competition. In Sections 5.2–5.4, we provide robustness checks with respect to information and cost assumptions: in Section 5.2, we

<sup>38</sup>The threshold discount factors for a second-price auction are then  $\hat{\delta}_{SPA}^{Stag} = \frac{\sqrt{r^4 - 3r^2 + 9} - r^2}{3 - r^2}$  and  $\hat{\delta}_{SPA}^{Sync} = \frac{\sqrt{3}}{\sqrt{r^2 + 3}}$ , whereas for a first-price auction they remain given by the expressions in footnote 26. It can be checked that, for  $s \in \{Sync, Stag\}$ ,  $\hat{\delta}_{SPA}^s > \hat{\delta}_{FPA}^s \iff r < \frac{1}{2}$ .

show that there is a greater risk of collusion when suppliers are more efficient; in Section 5.3, we consider the implications of suppliers having the same costs; and in Section 5.4, we assume that suppliers have symmetric information about their costs. In Section 5.5, we expand the set of available deterrence strategies to include asymmetric reserves. Finally, in Section 5.6, we consider the effects of requiring suppliers to register prior to a procurement in order to be eligible to participate.

## 5.1 Product market competition

We show here that, with an appropriate reinterpretation of our setting, our insights are relevant for Bertrand competition in product markets. Consider an environment with two product markets  $k \in \{1, 2\}$ , in which two large firms and a competitive fringe produce a homogeneous good and compete à la Bertrand. Demand is inelastic and normalized to unity (i.e.,  $D(p) = 1$ ), and firms have constant returns to scale. The large firms privately and independently draw their marginal costs from distribution  $G$ ; as before, their costs could either be market specific or the same in both markets. By contrast, the competitive fringe has deterministic and publicly known marginal cost,  $r < \bar{c}$ , which determines the collusive price.

In this setting, the competitive fringe obtains no profit and our analysis carries through by redefining  $\pi^m(c)$  as the monopoly payoff and  $\pi^c(c)$  as the competitive payoff per market for a firm with cost  $c$ . A market allocation scheme then assigns each market to a designated large firm, as follows: (i) the designated firm prices slightly below the competitive fringe’s marginal cost  $r$  when its own cost lies below it, and at cost otherwise; (ii) the non-designated firm prices slightly below  $r$ , but above the collusive price set by the designated firm, whenever its cost lies below it, and prices at cost otherwise. Under this market allocation, when a large firm’s cost  $c$  lies below  $r$ , the optimal deviation consists in slightly undercutting the designated firm’s target price. The analysis is then the same as in the first-price auction case of our baseline setup, generalizing to the present setting the insight that synchronized sales facilitates (initiation and sustainability of) collusion relative to simultaneous purchases.

## 5.2 Efficiency of suppliers

In order to analyze the effects of improvements in the suppliers’ cost distributions, consider the efficiency-parameterized distribution  $G_\sigma(c) \equiv 1 - [1 - G(c)]^\sigma$  with support  $[\underline{c}, \bar{c}]$ , where  $\sigma > 0$  measures the efficiency (or “strength”) of the cost distribution in the first-order stochastically dominated sense (the higher is the value of  $\sigma$ , the greater is the weight on low costs). We focus on the case of  $r = \bar{c}$  and show next that, as  $\sigma$  increases, the threshold

discount factor is eventually decreasing in  $\sigma$ , implying that the market is more susceptible to collusion.

The result is easiest to see for the case of synchronized purchasing. When  $r = \bar{c}$ , we have  $\bar{\pi}^n = 0$ , and so, depending on the auction format, the numerator in (13) is  $S_{SPA} = \pi^c(\underline{c})$  or  $S_{FPA} = \bar{c} - \underline{c}$ . As the efficiency of the suppliers' cost distributions increases,  $\pi^c(\underline{c})$  decreases because the payoff to a supplier with cost  $\underline{c}$  is increasingly constrained by competition from the rival. This means that the numerator in (13) is either decreasing or constant in  $\sigma$ , depending on the auction format. Because  $\bar{\pi}^m$  is increasing in  $\sigma$  and  $\bar{\pi}^c$  is decreasing in  $\sigma$  for  $\sigma$  sufficiently large (as the suppliers' payoffs are eventually driven to zero when competitors become extremely strong), the denominator in (13) is increasing in  $\sigma$ . It follows that the right side of (13), and hence the threshold discount factor, is decreasing in  $\sigma$  for  $\sigma$  sufficiently large. A similar, although more tedious, argument applies to the case of staggered purchasing. Intuitively, increasing efficiency increases the monopoly payoff and decreases the competitive payoff, making coordination increasingly attractive relative to competition. In addition, the deviation payoff of the worst type is not affected for a first-price auction and is reduced for a second-price auction as efficiency increases, which makes deviations less attractive.

Abusing notation by letting  $\delta_a^s(\sigma)$  denote the threshold discount factor for a given  $\sigma$ ,  $s \in \{Sync, Stag\}$  and  $a \in \{SPA, FPA\}$ , we have the following result:

**Proposition 8.** *For  $r = \bar{c}$ , increasing suppliers' efficiency facilitates collusion for sufficiently efficient suppliers. That is,  $\delta_a^s(\sigma)$  decreases in  $\sigma$  for  $\sigma$  sufficiently large for  $s \in \{Sync, Stag\}$  and  $a \in \{SPA, FPA\}$ .*

*Proof.* See Appendix A.6.

An implication of Proposition 8 is that there is likely a greater risk of collusion based on a market allocation from more efficient suppliers relative to less efficient ones. Thus, we obtain a version of the “topsy turvy principle” (see, e.g., Shapiro, 1989): the presence of efficient (low cost) suppliers offers the benefit of low prices under competition, but comes with the disadvantage of an increased risk of collusion.

### 5.3 Correlation between costs

In our baseline setting, suppliers are *ex ante* symmetric but *ex post* their costs differ with probability one because they are drawn independently from the same continuous distribution. It follows that allocating markets generates inefficiencies whenever both costs lie below the reserve because one market is then served by the supplier with the higher cost. To remove this inefficiency, we briefly consider here a variant in which costs

are perfectly correlated; that is, both suppliers face the same cost, drawn from the same distribution as before.

With perfectly correlated costs,  $\pi^c(c) = 0$  and  $\pi^n(c) = 0$ . The long-term and short-term stakes are otherwise the same as with independent draws. It follows, in particular, that  $S_{SPA} = 0$ . Hence, with a second-price auction, there are no profitable deviations and, for costs below the reserve, initiating collusion by bidding the reserve is always profitable. Further, as in the baseline setting: (i) collusion is easier with second-price auctions than with first-price auctions (indeed, it is always feasible with a second-price auction); and (ii) with first-price auctions, collusion is easier under staggered than under synchronous purchasing.

## 5.4 Symmetric information among suppliers

Finally, we show that similar insights apply when suppliers have independent costs but observe each other's costs. Competition then generates a profit equal to

$$\max \{0, \min \{r, c_{(2)}\} - c_{(1)}\}$$

for the supplier with the lower cost,  $c_{(1)}$ , and zero profit for the other supplier, whose cost is  $c_{(2)}$ . Perfect collusion can now be achieved without any communication by having the supplier with the higher cost bid the reserve price (in second-price auctions) or higher (in second-price and first-price auctions). Furthermore, whenever the less efficient supplier has a cost lower than the reserve, bidding the reserve (instead of bidding competitively) can signal *at no cost* its willingness to initiate such collusion.

This collusion thus generates the per-market industry monopoly profit

$$\max \{0, r - c_{(1)}\}$$

for the supplier with the lower cost, and zero profit for the other supplier. It is moreover always sustainable in case of second-price auctions: the more efficient supplier then bids at cost, and the other supplier—who faces a higher cost—cannot profitably deviate and undercut its rival. Hence, under both synchronous and staggered purchasing, suppliers can sustain efficient collusion and generate in this way an expected per-market industry monopoly profit equal to

$$\bar{\pi}_e^m = \mathbb{E}_{c_{(1)}} [\max \{0, r - c_{(1)}\}],$$

where the subscript “*e*” refers to “efficient collusion.”

In the case of first-price auctions, the less efficient supplier can instead profitably

deviate whenever its cost lies below the reserve. The best deviation is then to undercut slightly the reserve price, and it is more profitable the lower is the cost of less efficient supplier. The most profitable deviation thus obtains when  $c_{(2)}$  is arbitrarily close to  $\underline{c}$ , in which case it yields

$$\pi^m(\underline{c}) = r - \underline{c}.$$

The deviation then triggers a reversal to competition, in which each firm's expected profit is

$$\bar{\pi}_e^c = \frac{1}{2} \mathbb{E}_{c_{(1)}, c_{(2)}} [\max \{0, \min \{r, c_{(2)}\} - c_{(1)}\}].$$

Under synchronous purchasing, the most profitable deviation consists in deviating in both markets, and it triggers punishments two periods later; collusion is therefore sustainable if and only if

$$2\pi^m(\underline{c}) + \frac{\delta^2}{1 - \delta^2} 2\bar{\pi}_e^c \leq \frac{\delta^2}{1 - \delta^2} \bar{\pi}_e^m,$$

which is equivalent to

$$\frac{\delta^2}{1 - \delta^2} \geq \frac{2\pi^m(\underline{c})}{\bar{\pi}_e^m - 2\bar{\pi}_e^c}. \quad (22)$$

By contrast, under staggered purchasing, the deviation takes place in a single market, and triggers punishment in the next period. Hence, collusion is sustainable if and only if

$$\pi^m(\underline{c}) + \frac{\delta}{1 - \delta} \bar{\pi}_e^c \leq \frac{\delta}{1 - \delta} \frac{\bar{\pi}_e^m}{2},$$

or, equivalently,

$$\frac{\delta}{1 - \delta} \geq \frac{2\pi^m(\underline{c})}{\bar{\pi}_e^m - 2\bar{\pi}_e^c}. \quad (23)$$

Therefore, as in the case of perfectly correlated costs, (i) collusion is again always feasible with second-price auctions; and (ii) with first-price auctions, collusion is easier under staggered than under synchronous purchasing.

Because these results show that complete information can increase the expected profit from a market allocation, they have implications for suppliers' incentives to communicate regarding their costs. In the presence of such communication, and assuming verifiability, efficient collusion is sustainable with second-price auctions, but might not be with first-price auctions. An implication is that suppliers might have a greater incentive to communicate if the auction is a second-price auction rather than a first-price auction.

*Remark: on efficient collusion versus market allocation.* The above form of collusion, in which the firm with the lower cost supplies both markets, is efficient and thus gener-

ates greater profit than a market allocation (which yields the same profits, regardless of whether suppliers observe each other’s costs). In the case of second-price auctions, it is moreover easier to sustain—indeed, it is always sustainable. In the case of first-price auctions, however, the above efficient collusion can be more difficult to sustain than a market allocation. The reason depends on the purchasing sequence. In the case of synchronous purchasing, under efficient collusion, the higher-cost supplier can now deviate and obtain the monopoly profits in *both* markets. This makes the deviation more profitable, and can offset the impact of the greater benefit from future collusion.<sup>39</sup> In the case of staggered purchasing, deviations are equally profitable in both forms of collusion. However, with efficient collusion, each supplier has half a chance of obtaining the monopoly profit in every period. By contrast, with a market allocation, the relevant deviant supplier is precisely the one that is designated to win in the following period. As a result, with a market allocation, the benefit of future collusion comes sooner, which can again offset the fact that this benefit is lower than with efficient collusion.<sup>40</sup> Whenever this is the case, however, firms could instead settle for “less efficient” collusion, in which the lower-cost firm supplies both markets only when the higher cost exceeds some threshold, and rely on a market allocation when both costs lie below this threshold.

## 5.5 Deterrence using asymmetric reserves

In this extension, we consider the possibility of asymmetric reserves across the two markets in the context of uniformly distributed costs and with  $v = 1$ . As will be clear, the qualitative results continue to hold as long as the long-term stake is monotone in  $\delta$  for reserves in the relevant range. A characterization of the distributional assumptions required to guarantee this condition is beyond the scope of the paper.

Without loss of generality, we assume  $r_1 \leq r_2$ . We first focus on the case of a single buyer operating in both markets, before discussing the coordination issues that can arise when there are two separate buyers. We consider here the case of synchronized purchasing and relegate to Appendix B the case of staggered purchasing, which has greater complexity but yields the same qualitative insights: the buyer optimally adjusts reserves to deter collusion for an intermediate range of discount factors and accommodates collusion for sufficiently high discount factors.

When the suppliers face asymmetric reserves, they may want to avoid having the same supplier systematically facing the lower reserve, as they would under a market allocation

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<sup>39</sup>For example, if  $G(c) = c^a$  for  $a > 0$  with support  $[0, 1]$  and  $r = 1$ , then the threshold discount factor defined by (22) is greater than  $\delta_{FPA}^{Sync}$  if and only if  $a < 1$ .

<sup>40</sup>For example, if  $G(c) = c^a$  for  $a > 0$  with support  $[0, 1]$  and  $r = 1$ , then the threshold discount factor defined by (23) is greater than  $\delta_{FPA}^{Stag}$  if and only if  $a < 1/16(5 + \sqrt{217}) = 1.23$ .

scheme. Therefore, for synchronized purchasing, a relevant and natural alternative collusive strategy is a rotation, whereby the suppliers switch their designated markets in each round of purchasing; as before, we assume that any deviation in either market triggers a reversal to competition in both markets. The long-term stake for the supplier designated for market  $i$ , who will rotate to market  $j$  in two periods and then back to market  $i$  in four periods, where  $i \neq j \in \{1, 2\}$ , is

$$L_{Rot}^{Sync}(\delta, r_i, r_j) \equiv \frac{\delta^2}{1 - \delta^4} [\bar{\pi}^m(r_j) - \bar{\pi}^c(r_j)] - \frac{\delta^2}{1 - \delta^4} [\bar{\pi}^c(r_i) - \bar{\pi}^n(r_i)] \\ + \frac{\delta^4}{1 - \delta^4} [\bar{\pi}^m(r_i) - \bar{\pi}^c(r_i)] - \frac{\delta^4}{1 - \delta^4} [\bar{\pi}^c(r_j) - \bar{\pi}^n(r_j)].$$

A rotation is sustainable if and only if:

$$L_{Rot}^{Sync}(\delta, r_1, r_2) \geq S_a(r_2) \quad \text{and} \quad L_{Rot}^{Sync}(\delta, r_2, r_1) \geq S_a(r_1). \quad (24)$$

The joint profit of the suppliers and the payoff of the buyer are the same under a rotation and a market allocation,<sup>41</sup> but a rotation is easier to sustain.<sup>42</sup> Intuitively, collusion is easier to sustain because both suppliers alternately enjoy the benefit of collusion in the more profitable market (i.e., the market with the higher reserve). Further, a rotation involving both markets is easier to sustain than a rotation in just one market, combined with competitive bidding in the other market, for the relevant range of discount factors.<sup>43</sup> Guided by this, in what follows, we restrict attention to collusion based on a rotation in both markets.

Figure 5 shows the suppliers' conduct for second-price auctions with different combi-

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<sup>41</sup>The suppliers can moreover share the expected profit from collusion by randomizing over the initial market designation.

<sup>42</sup>When  $r_2 > r_1$ , the relevant long-term stake for a rotation is  $L_{Rot}^{Sync}(\delta, r_1, r_2)$  and for a market allocation is  $L^{Sync}(\delta, r_1, r_2)$  (defined in footnote 33), where, using the assumption that costs are uniformly distributed on  $[0, 1]$ ,

$$L_{Rot}^{Sync}(\delta, r_1, r_2) - L^{Sync}(\delta, r_1, r_2) = \frac{(r_2^3 - r_1^3) \delta^2}{2(1 - \delta^4)},$$

which is positive for  $r_2 > r_1$ , implying that a rotation is easier to sustain than a market allocation.

<sup>43</sup>The long-term stake for a rotation involving only, say, market 2 is  $L_{Rot}^{Sync}(\delta, \underline{c}, r_2)$ . Note that

$$L_{Rot}^{Sync}(\delta, r_1, r_2) - L_{Rot}^{Sync}(\delta, \underline{c}, r_2) = \frac{\delta^2}{1 - \delta^4} [\delta^2 \bar{\pi}^m(r_1) + \bar{\pi}^n(r_1) - (\delta^2 + 1) \bar{\pi}^c(r_1)],$$

which is positive for discount factors that are sufficiently close to one when the market is at risk for collusion (i.e., under condition (5)). It follows that for sufficiently large discount factors, a rotation in both markets is easier to sustain than a rotation in just one. For uniformly distributed costs, this holds for all relevant discount factors, i.e., for all discount factors sufficiently large that a rotation is sustainable for reserves  $(r_1, r_2) = (r^{Comp}, r^{Comp})$ .

nations of reserves, along with the optimal competitive, collusive, and deterrence reserves for  $\delta = 0.93$ . This figure illustrates a case in which collusion is not blocked—at the optimal competitive reserves, collusion is sustainable. Furthermore, starting from the optimal symmetric deterrence reserve, it is possible to increase both reserves (asymmetrically) and still deter collusion; it follows that, conditional on deterring collusion, it is optimal to adopt asymmetric reserves.

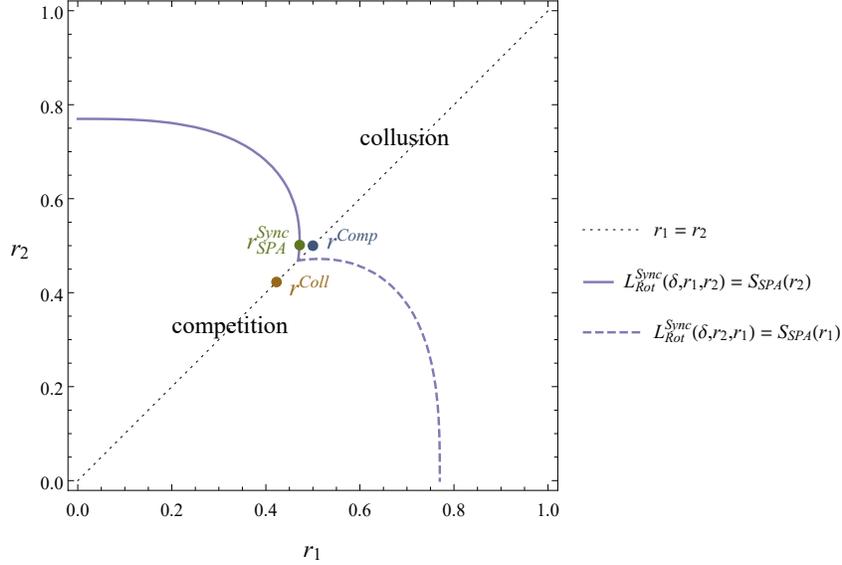


Figure 5: Suppliers' conduct as a function of the reserves for synchronized purchasing and second-price auction. Assumes that costs are uniformly distributed over  $[0, 1]$ ,  $v = 1$ , and  $\delta = 0.93$ .

More generally, let  $\underline{\delta}_a^{Sync}(v)$  be the largest discount factor such that there is competition at reserves of  $r_1 = r_2 = r^{Comp}(v)$ ; for  $\delta \leq \underline{\delta}_a^{Sync}(v)$ , setting both reserves equal to  $r^{Comp}(v)$  is clearly optimal. For  $\delta > \underline{\delta}_a^{Sync}(v)$ , define

$$(r_{1,a}^{Sync}(\delta), r_{2,a}^{Sync}(\delta)) \in \arg \max_{(r_1, r_2) \in [\underline{c}, \min\{\bar{c}, v\}]^2} \{U^{Comp}(r_1) + U^{Comp}(r_2) \mid r_1 \leq r_2, (24) \text{ fails}\}.$$

Reserves slightly below  $(r_{1,a}^{Sync}(\delta), r_{2,a}^{Sync}(\delta))$  optimally deter collusion (they correspond to  $r_{SPA}^{Sync}$  in Figure 5). It remains to identify conditions under which it is optimal for the buyer to use these deterrence reserves. Define  $\bar{\delta}_a^{Sync}(v)$  to be the largest discount factor such that deterrence is optimal, that is,

$$\begin{aligned} \bar{\delta}_a^{Sync}(v) &\equiv \max \left\{ \delta \in [0, 1] \mid U^{Comp}(r_{1,a}^{Sync}(\delta)) + U^{Comp}(r_{2,a}^{Sync}(\delta)) \geq 2U^{Coll}(r^{Coll}(v)) \right\} \\ &= \min \left\{ \delta \in [0, 1] \mid U^{Comp}(r_{1,a}^{Sync}(\delta)) + U^{Comp}(r_{2,a}^{Sync}(\delta)) \leq 2U^{Coll}(r^{Coll}(v)) \right\}, \end{aligned}$$

where the equality uses the monotonicity of  $L_{Rot}^{Sync}(\delta, r_1, r_2)$  in  $\delta$ .

It then follows that for  $a \in \{SPA, FPA\}$  and  $\delta \in (0, 1)$ , the buyer's optimal reserves for synchronized purchasing are

$$(\rho_{1,a}^{Sync}(\delta), \rho_{2,a}^{Sync}(\delta)) \equiv \begin{cases} (r^{Comp}(v), r^{Comp}(v)) & \text{if } \delta \leq \underline{\delta}_a^{Sync}(v), \\ (r_{1,a}^{Sync}(\delta), r_{2,a}^{Sync}(\delta)) & \text{if } \delta \in (\underline{\delta}_a^{Sync}(v), \bar{\delta}_a^{Sync}(v)], \\ (r^{Coll}(v), r^{Coll}(v)) & \text{otherwise.} \end{cases} \quad (25)$$

We illustrate the asymmetric optimal reserves for synchronized purchasing and second-price auctions as a function of  $\delta$  in Figure 6. Figure 6(a) shows the optimized profit for the buyer under the optimal asymmetric reserves, and Figure 6(b) shows the optimal reserves. As highlighted by Figure 6(b), the optimal deterrence reserves are asymmetric, and the optimal reserves are discontinuous in the discount factor as it increases from the region of deterrence to the region of accommodation.

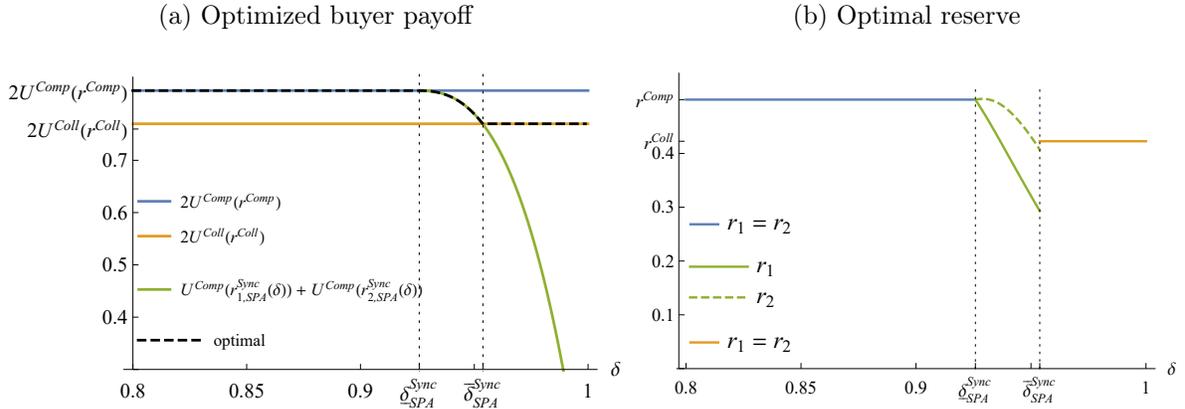


Figure 6: Illustration of the buyer's expected payoff under the optimal asymmetric reserves and the optimal asymmetric reserves themselves, assuming  $r_1 \leq r_2$ , as functions of the discount factor, under synchronized purchasing and second-price auctions. Assumes that costs are uniformly distributed over  $[0, 1]$  and  $v = 1$ . The point of discontinuity in Panel (b) occurs at  $\delta \cong 0.954$ .

## Independent buyers and the possibility of coordination failure

The possibility of asymmetric reserves raises the prospect of coordination issues when there are two different buyers, one in each market. For example, consider the case of synchronized purchasing and second-price auctions, and suppose that the two buyers simultaneously set their reserves, which are then fixed for all time.<sup>44</sup> Figure 7 shows the

<sup>44</sup>The possibility of dynamic adjustments, of course, raises additional complexities.

buyers' best responses (i.e., each buyer's optimal reserve as a function of the other buyer's reserve), for two different discount factors, maintaining our assumptions that costs are uniformly distributed over  $[0, 1]$  and  $v = 1$ .

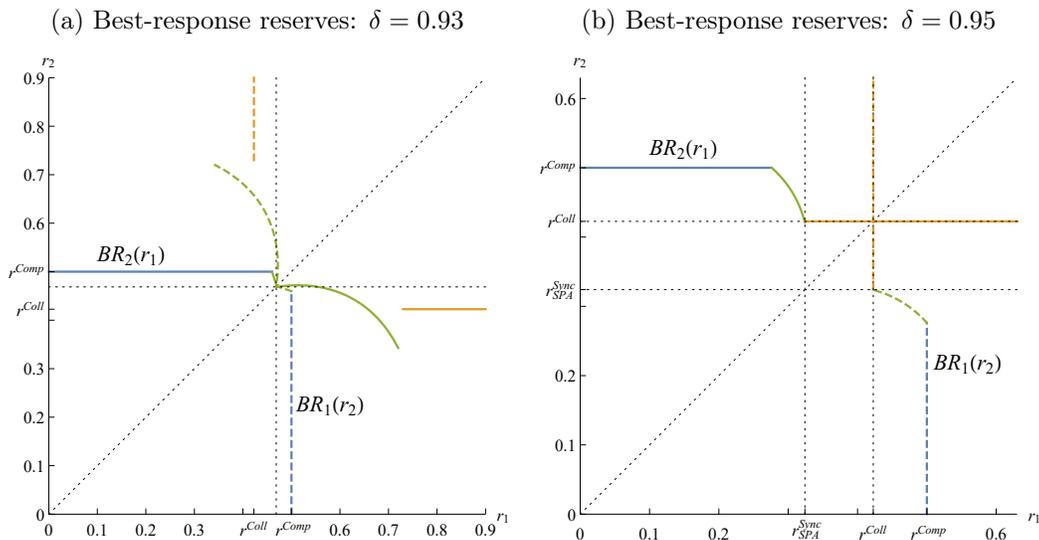


Figure 7: Best-response reserves for synchronized purchasing and second-price auctions. Assumes that costs are uniformly distributed over  $[0, 1]$  and  $v = 1$ . Discount factors are as indicated. In Panel (a), the unlabelled dotted line is  $r_{SPA}^{Sync}(\delta)$  for  $\delta = 0.93$ . In Panel (b), although it is difficult to discern in the graphic, the best response function jumps up from 0.421 to 0.423 as the best response moves from the region of deterrence to the region of accommodation.

For both selected values of the discount factor, a single buyer setting the reserves for both markets would choose asymmetric deterrence reserves. As shown in Figure 7(a), when  $\delta = 0.93$ , the Nash equilibrium reserves are symmetric deterrence reserves—they are equal to the best symmetric deterrence reserve for a single buyer. Thus, deterrence continues to occur, but coordination failure results in the use of symmetric reserves, rather than asymmetric ones, which are better for the buyers in aggregate. The coordination failure harms the buyers, as it reduces their joint profit, but it also harms suppliers and society, because the Nash equilibrium reserve is lower than both optimal (asymmetric) deterrence reserves.<sup>45</sup> Given that collusion is deterred, the suppliers prefer that it be done optimally. In Figure 7(b), we assume that  $\delta = 0.95$ , which remains in the range where a single buyer would optimally deter collusion, as indicated in Figure 6(b). In this case, the buyers choose  $r^{Coll}$  in the Nash equilibrium. There is complete coordination failure in that collusion occurs precisely because the buyers fail to coordinate their procurement

<sup>45</sup>For  $\delta = 0.93$ , the optimal symmetric deterrence reserve (and Nash equilibrium reserve) is 0.4684, and the optimal (asymmetric) deterrence reserves are, as illustrated in Figure 6,  $(r_{1,SPA}^{Sync}(0.93), r_{2,SPA}^{Sync}(0.93)) = (0.4717, 0.5013)$ .

mechanisms. Production under collusion is less efficient than under competition, but the collusive reserve is sufficiently close to the higher of the optimal asymmetric deterrence reserves that the overall probability of trade is increased.<sup>46</sup> As a result, the coordination failure reduces total buyer surplus but increases both total supplier surplus and social surplus. The optimal deterrence reserves are sufficiently aggressive that society is better off with collusion.

Interestingly, this possibility of coordination failure provides a rationale for the popular view that large buyers are less prone to be victims of collusion.<sup>47</sup> In our setup, a single, large buyer would internalize the benefits from coordinating the reserves across the two markets. To the best of our knowledge, this is the first formalization of this notion. It apparently contrasts with Loertscher and Marx (2019b), who show that endowing a buyer with buyer power makes covert collusion more attractive relative to a merger because the merger is a public event and causes the powerful buyer to react in a way that is detrimental to the merging suppliers. In light of footnote 23, the way to reconcile these statements is that *powerful* and *large* buyers are distinct things. Our independent buyers are, by our assumptions, very powerful because they can commit to a binding reserve forever. Yet, lacking size (or coordination), all this power is not necessarily enough to deter collusion.

## 5.6 Supplier registration

In some cases, buyers utilize a registration process for potential suppliers in advance of collecting bids, with the set of registered suppliers being observable at the time that bids are requested (Marshall and Marx, 2009). In this section, we consider the effects of such a registration process on the sustainability and initiation of a market allocation.

We first note that the long-term stakes are not affected by supplier registration, so we can focus on the effects of registration on the short-term stakes. Registration has no effect on the short-term stakes either in the “baseline market allocation” in which, as above, the suppliers register in both markets and each supplier bids no less than the reserve in its rival’s allocated market. It follows that the introduction of registration does not make the markets less vulnerable to collusion.

The suppliers could, however, take advantage of the registration scheme. For example, they could use a collusive scheme specifying that once a market allocation is initiated, a supplier does not register in its rival’s market, and if a supplier deviates by registering in its rival’s market, then bidding reverts to noncooperative bidding forever after. We refer to this as a “registration-based market allocation.” Under a registration-based market

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<sup>46</sup> For  $\delta = 0.95$ , the optimal deterrence reserves are  $(r_{1,SPA}^{Sync}(0.95), r_{2,SPA}^{Sync}(0.95)) = (0.3211, 0.4311)$ , compared with  $r^{Coll} = 0.4227$ .

<sup>47</sup>See, for example, Carlton and Israel (2011) for an expression of this view.

allocation, a supplier's expected payoff in its rival's market is zero ( $\pi^n(c) = 0$ ), and a supplier's deviation payoff is the competitive payoff, regardless of whether the auction is a second-price or first-price auction. Thus,  $\bar{S}_{SPA} = \bar{S}_{FPA} = \pi^c(\underline{c})$ . Further, if costs are only realized after registration, then the relevant deviation payoff is the expected deviation payoff, implying that  $\bar{S}_{SPA} = \bar{S}_{FPA} = \bar{\pi}^c$ . These differences are summarized in Table 2.

Table 2: Effects of registration

setting and market allocation	cost realization relative to registration	short-term stake	
		SPA	FPA
without registration	–	$\pi^c(\underline{c}) - \pi^n(\underline{c})$	$\pi^m(\underline{c}) - \pi^n(\underline{c})$
with registration			
baseline mkt allocation	before or after	$\pi^c(\underline{c}) - \pi^n(\underline{c})$	$\pi^m(\underline{c}) - \pi^n(\underline{c})$
registration-based mkt allocation	before	$\pi^c(\underline{c})$	$\pi^c(\underline{c})$
registration-based mkt allocation	after	$\bar{\pi}^c$	$\bar{\pi}^c$

With registration, the short-term stake is sometimes, but not always, reduced relative to the case without registration. Colluding suppliers can always do just as well with registration as without (e.g., by using the baseline market allocation) and are able to sustain collusion for a greater range of discount factors by using a registration-based market allocation in some cases. For example, the short-term stake is reduced under a registration-based market allocation if  $r = \bar{c}$ , so that  $\pi^n(\underline{c}) = 0$ , and if either the auction format is first price or costs are realized only after registration. Turning briefly to the initiation of a market allocation, initiation based on bidding the reserve involves the same tradeoffs with or without registration, which implies that the presence of registration cannot make initiation more difficult. This gives us the following result:

**Proposition 9.** *Regardless of the timing of purchasing (synchronized or staggered), registration never makes a market less vulnerable to collusion and makes a market more vulnerable to collusion if  $r = \bar{c}$  and if either the auction format is first price or costs are realized only after registration.*

These results are consistent with conclusions in the literature that preauction transparency in the form of transparent registration can increase susceptibility to collusion (see, e.g., Marshall and Marx, 2009).

## 6 Conclusion

We study collusion based on market allocations and show that, contrary to some prevailing beliefs, the presence of staggered purchasing does not make collusion more difficult to sustain or initiate, relative to synchronized purchasing. We show that market allocation is profitable in some but not all markets—while suppliers engaged in a market allocation benefit from the reduction in competition, they are harmed by the resulting inefficiency. Markets are more at risk if second-price auctions are used rather than first-price auctions (unless only the award price is observable and the reserve price is sufficiently low), if buyers are not powerful, in the sense that they are unable to commit to reserve prices, and if suppliers are more efficient in the sense that they have better cost distributions. Supplier registration also makes a market more vulnerable to collusion. As defensive measures, buyers can strategically set (possibly secret or asymmetric) reserve prices, withhold information on bids and award price, and impose longer contract duration. Some of these measures, however, increase the inefficiency of procurement and suffer from problems of coordination among buyers. Our results on the impact on collusion of tendering timing and auction format are robust to changes in information and cost environments, such as when suppliers have the same costs or they have symmetric information about their costs.

Our analysis also sheds light on the early stages of the collusive process, in which suppliers initiate their agreements. Antitrust cases have been informative about various ways for suppliers to coordinate explicitly, for example through pre-auction meetings or emails, but little is known regarding how suppliers convey their intention to coordinate in a tacit manner. This has led some authorities to dismiss the risk of collusion (see the quote from the judge in the AT&T–Time Warner merger). Our analysis shows instead that initiation is feasible whenever collusion is sustainable, suggesting that tacit collusion is a more severe problem than currently recognized.

The insights developed in this paper generate a number of implications for public and private procurement. For example, at a time where numerous countries are progressively opening up public services to competitive tendering, our paper offers a further reason, beyond alleviating the risk and the cost of monopolization (Cabral, 2017; Iossa et al., 2019), in favour of synchronized tendering for the provision of public services. Moreover, the possibility of coordination failure among buyers in adopting defensive strategies provides a rationale (beyond economies of scope and buyer power) for centralizing purchasing units. In public procurement, this can be done by setting up national or regional procurement authorities that operate on behalf of local offices. In private procurement, it can be achieved by managing some repeated purchases at central rather than division level.

A number of issues could be explored in future research. First, the analysis could be extended to stochastic bidder selection schemes. These have been proven to be individu-

ally rational under certain conditions (Loertscher and Marx, 2019a) and more difficult to detect. Along the same lines, the analysis could investigate alternative defensive measures, such as the ones advocated by the OECD (2009), whereby each buyer changes the size and timing of procurement unpredictably. Moreover, further research on the initiation of collusion would be valuable.

# A Proofs

## A.1 Proof of Lemma 1

When  $r = \bar{c}$ , we have  $\bar{\pi}^n = 0$ , and so we can write (5) as

$$\int_{\underline{c}}^{\bar{c}} G(c) dc > 2 \int_{\underline{c}}^{\bar{c}} G(c) [1 - G(c)] dc.$$

Using integration by parts, this amounts to

$$\bar{c} - \int_{\underline{c}}^{\bar{c}} cg(c) dc > -2 \int_{\underline{c}}^{\bar{c}} c [1 - 2G(c)] g(c) dc = \int_{\underline{c}}^{\bar{c}} c 2G(c) g(c) dc - \int_{\underline{c}}^{\bar{c}} c 2 [1 - G(c)] g(c) dc,$$

which is equivalent to

$$\bar{c} - \mathbb{E}_c[c] > \mathbb{E}_c[c_{(2)}] - \mathbb{E}_c[c_{(1)}].$$

Turning to the second statement in the proof, for any reserve  $r \in [\underline{c}, \bar{c}]$ , the impact of collusion on total profit is equal to:

$$\begin{aligned} \Delta(r) &\equiv \bar{\pi}^m(r) + \bar{\pi}^n(r) - 2\bar{\pi}^c(r) \\ &= \int_{\underline{c}}^r [2G(c) - G(r)] G(c) dc. \end{aligned}$$

The market is at risk (i.e., collusion is strictly profitable) if and only if  $\Delta(r) > 0$ . We have:

$$\begin{aligned} \Delta'(r) &= G^2(r) - \int_{\underline{c}}^r g(r) G(c) dc \\ &= G(r) \int_{\underline{c}}^r g(c) dc - \int_{\underline{c}}^r g(r) G(c) dc \\ &= \int_{\underline{c}}^r G(r) G(c) \left[ \frac{g(c)}{G(c)} - \frac{g(r)}{G(r)} \right] dc \\ &> 0, \end{aligned}$$

where the inequality stems from the monotonicity of the reverse hazard rate. As  $\Delta(\underline{c}) = 0$ , it follows that  $\Delta(r) > 0$  for any  $r \in (\underline{c}, \bar{c}]$ . ■

## A.2 Proof of Proposition 3

It suffices to show that the right-hand sides of conditions (13) and (14) are decreasing functions of  $r$  for  $r$  sufficiently close to  $\underline{c}$ . Because the right side of (13) is a particular

case of (14) (namely, for  $\delta = 1$ ), we can focus on the latter, which can be written as

$$RHS_a(r) \equiv \frac{N_a(r)}{D(r)},$$

where

$$N_a(r) \equiv S_a = \begin{cases} (r - \underline{c})G(r) - \int_{\underline{c}}^r G(c)dc, & \text{if } a = SPA, \\ (r - \underline{c})G(r), & \text{if } a = FPA, \end{cases}$$

and

$$D(r) \equiv \bar{\pi}^m - \bar{\pi}^c + \delta(\bar{\pi}^n - \bar{\pi}^c) = \int_{\underline{c}}^r [(1 + \delta)G(c) - \delta G(r)]G(c)dc.$$

We thus have

$$RHS'_a(r) = \frac{\hat{N}_a(r)}{\hat{D}(r)},$$

where

$$\hat{N}_a(r) \equiv D(r)N'_a(r) - D'(r)N_a(r) \quad \text{and} \quad \hat{D}(r) \equiv D^2(r).$$

Using

$$N'_a(r) = \begin{cases} (r - \underline{c})g(r), & \text{if } a = SPA, \\ G(r) + (r - \underline{c})g(r), & \text{if } a = FPA, \end{cases}$$

$$N''_a(r) = \begin{cases} g(r) + (r - \underline{c})g'(r), & \text{if } a = SPA, \\ 2g(r) + (r - \underline{c})g'(r), & \text{if } a = FPA, \end{cases}$$

and

$$D'(r) = G^2(r) - \delta g(r) \int_{\underline{c}}^r G(c)dc,$$

$$D''(r) = (2 - \delta)G(r)g(r) - \delta g'(r) \int_{\underline{c}}^r G(c)dc,$$

$$D'''(r) = (2 - \delta)g^2(r) + 2(1 - \delta)G(r)g'(r) - \delta g''(r) \int_{\underline{c}}^r G(c)dc,$$

yields

$$N_a(\underline{c}) = N'_a(\underline{c}) = 0 < N''_a(\underline{c}) \quad \text{and} \quad D(\underline{c}) = D'(\underline{c}) = D''(\underline{c}) = 0 < D'''(\underline{c}).$$

Building on this, we have:

$$\begin{aligned}
\hat{N}'_a(r) &= D(r) N''_a(r) - D''(r) N_a(r), \\
\hat{N}''_a(r) &= D(r) N'''_a(r) + D'(r) N''_a(r) - D''(r) N'_a(r) - D'''(r) N_a(r), \\
\hat{N}'''_a(r) &= D(r) N''''_a(r) + 3D'(r) N'''_a(r) + 2D''(r) N''_a(r) - 2D'''(r) N'_a(r) \\
&\quad - 3D''''(r) N_a(r) - D''''''(r) N_a(r),
\end{aligned}$$

and

$$\begin{aligned}
\hat{D}'(r) &= 2D(r) D'(r), \\
\hat{D}''(r) &= 2D(r) D''(r) + 2[D'(r)]^2, \\
\hat{D}'''(r) &= 2D(r) D'''(r) + 6D'(r) D''(r), \\
\hat{D}''''(r) &= 2D(r) D''''(r) + 8D'(r) D'''(r) + 6[D''(r)]^2, \\
\hat{D}''''''(r) &= 2D(r) D''''''(r) + 10D'(r) D''''(r) + 20D''(r) D'''(r), \\
\hat{D}''''''''(r) &= 2D(r) D''''''''(r) + 12D'(r) D''''''(r) + 30D''(r) D''''(r) + 20[D'''(r)]^2,
\end{aligned}$$

which yields:

$$\begin{aligned}
\hat{N}_a(\underline{c}) &= \hat{N}'_a(\underline{c}) = \hat{N}''_a(\underline{c}) = 0 > \hat{N}'''_a(\underline{c}) = -2D'''(\underline{c}) N''_a(\underline{c}), \\
\hat{D}(\underline{c}) &= \hat{D}'(\underline{c}) = \hat{D}''(\underline{c}) = \hat{D}'''(\underline{c}) = \hat{D}''''(\underline{c}) = \hat{D}''''''(\underline{c}) = 0 < \hat{D}''''''''(\underline{c}) = 20[D'''(\underline{c})]^2.
\end{aligned}$$

Using Taylor expansions then leads to:

$$\lim_{r \rightarrow \underline{c}} RHS'_a(r) = \lim_{r \rightarrow \underline{c}} \frac{\hat{N}_a(r)}{\hat{D}(r)} = \lim_{r \rightarrow \underline{c}} \frac{\hat{N}'''_a(\underline{c}) \frac{(r-\underline{c})^3}{3!}}{\hat{D}''''''(\underline{c}) \frac{(r-\underline{c})^6}{6!}} = \frac{6! \hat{N}'''_a(\underline{c})}{3! \hat{D}''''''(\underline{c})} \lim_{r \rightarrow \underline{c}} \frac{1}{(r-\underline{c})^3} = -\infty.$$

Because  $\lim_{r \rightarrow \underline{c}} RHS'_a(r) = -\infty$ , there exists  $\hat{r} > \underline{c}$  such that  $RHS'_a(r) < 0$  for  $r \leq [\underline{c}, \hat{r}]$ .

To show that the critical discount factors tend to 1 as  $r$  tends to  $\underline{c}$ , we first recall that these thresholds are determined by solving (13) and (14) with equality. In the case of synchronized purchasing, the right-hand side (13) increases from 0 to  $+\infty$  as  $\delta$  increases from 0 to 1, whereas the right-hand side is given by  $RHS_a(r; \delta = 1)$ . Using again Taylor expansions and noting that  $D'''(\underline{c}; \delta = 1) = g^2(\underline{c}) = N''_{SPA}(\underline{c}) \leq N''_a(\underline{c})$ , we obtain that this right-hand side satisfies:

$$\lim_{r \rightarrow \underline{c}} RHS_a(r; \delta = 1) = \lim_{r \rightarrow \underline{c}} \frac{N_a(r)}{D(r; \delta = 1)} = \lim_{r \rightarrow \underline{c}} \frac{N''_a(\underline{c}) \frac{(r-\underline{c})^2}{2!}}{D'''(\underline{c}; \delta = 1) \frac{(r-\underline{c})^3}{3!}} \geq 3 \lim_{r \rightarrow \underline{c}} \frac{1}{(r-\underline{c})^3} = +\infty.$$

In the case of staggered purchasing, the right-hand side (14) increases again from 0 to

$+\infty$  as  $\delta$  increases from 0 to 1, whereas the right-hand side satisfies, using  $D'''(\underline{c}; \delta) \leq 2g^2(\underline{c}) = 2N''_{SPA}(\underline{c}) \leq 2N''_a(\underline{c})$ :

$$\lim_{r \rightarrow \underline{c}} RHS_a(r; \delta) = \lim_{r \rightarrow \underline{c}} \frac{N_a(r)}{D(r; \delta)} = \lim_{r \rightarrow \underline{c}} \frac{N''_a(\underline{c}) \frac{(r-\underline{c})^2}{2!}}{D'''(\underline{c}; \delta) \frac{(r-\underline{c})^3}{3!}} \geq \frac{3}{2} \lim_{r \rightarrow \underline{c}} \frac{1}{(r-\underline{c})^3} = +\infty.$$

It follows that, in both cases, the critical discount factor tends to 1 as  $r$  tends to  $\underline{c}$ . ■

### A.3 Proof of Proposition 4

By virtue of the optimality of the reserves, we have

$$U^{Comp}(v, r^{Comp}(v)) \geq U^{Coll}(v, r^{Comp}(v)) \quad \text{and} \quad U^{Comp}(v, r^{Coll}(v)) \leq U^{Coll}(v, r^{Coll}(v)).$$

Combining these with (18) yields

$$\int_{\underline{c}}^{r^{Coll}(v)} G^2(x) dx \leq \int_{\underline{c}}^{r^{Comp}(v)} G^2(x) dx,$$

which implies that

$$r^{Coll}(v) \leq r^{Comp}(v).$$

Thus, a revealed preference argument shows that the optimal reserve is weakly more aggressive when facing collusion. Furthermore, we have:

$$\left. \frac{\partial U^{Comp}}{\partial r}(v, r) \right|_{r=r^{Coll}(v)} = \left. \frac{\partial U^{Coll}}{\partial r}(v, r) \right|_{r=r^{Coll}(v)} + G^2(r^{Coll}(v)) = G^2(r^{Coll}(v)) > 0.$$

It follows that  $r^{Coll}(v) < r^{Comp}(v)$ . ■

### A.4 Proof of Proposition 5

We already established in the text that collusion is blockaded when  $v < \underline{v}$  and that deterrence is optimal when  $\underline{v} \leq v \leq \bar{v}$ . To complete the proof, we now suppose that  $v > \bar{v}$ , in which case the relevant choice is between accommodating collusion, by choosing  $r^{Coll}(v)$ , or deterring it, by choosing a more aggressive reserve price slightly below  $r_a^s(\delta)$ . Let

$$\Delta U(v) \equiv U^{Coll}(v, r^{Coll}(v)) - U^{Comp}(v, r_a^s(\delta))$$

denote the payoff difference attached to these two options. Using the optimality of  $r^{Coll}(v)$  and (18), we have:

$$\Delta U(v) = \max_r (v - r)\hat{G}(r) - (v - r_a^s(\delta))\hat{G}(r_a^s(\delta)) - B(r_a^s(\delta)),$$

where  $B(r)$  is the competition benefit defined by (18). It follows that this difference strictly increases with  $v$ :

$$\Delta U'(v) = \hat{G}(\hat{r}) - \hat{G}(r_a^s(\delta)) > 0.$$

Furthermore, for  $v = \bar{v}$ , we have  $r^{Coll}(v) = r_a^s(\delta)$  and thus:

$$\Delta U(\bar{v}) = -B(r_a^s(\delta)) < 0.$$

By contrast, as  $v$  goes to infinity,  $r^{Coll}(v)$  tends to  $\bar{c}$ ; hence, there exists  $\epsilon > 0$  such that  $\hat{G}(r^{Coll}(v)) > \hat{G}(r_a^s(\delta)) + \epsilon$  for  $v$  large enough. Therefore, we have:

$$\begin{aligned} \Delta U(v) &> (v - \bar{c})[\hat{G}(r_a^s(\delta)) + \epsilon] - (v - r_a^s(\delta))\hat{G}(r_a^s(\delta)) - B(r_a^s(\delta)) \\ &= (v - \bar{c})\epsilon - (\bar{c} - r_a^s(\delta))\hat{G}(r_a^s(\delta)) - B(r_a^s(\delta)), \end{aligned}$$

where the first inequality relies on  $r^{Coll}(v) < \bar{c}$  and the last right-hand side goes to infinity with  $v$ . It follows that there exists a unique threshold  $\hat{v}$  for which  $\Delta U(\hat{v}) = 0$ , which concludes the proof. ■

## A.5 Proof of Proposition 6

Fix  $T$  and  $\tau \in \{1, \dots, T\}$ . If  $\tau = 1$ , purchases are perfectly synchronous and the analysis is the same as in the baseline model, replacing  $\delta^2$  with  $\delta^T$ . The long-term stake is therefore given by  $L^{Sync}(\delta^{T/2})$ . We now focus on asynchronous tenders (i.e.,  $\tau \in \{2, \dots, T\}$ ) and consider a tender taking place in a given period  $t$ , in which supplier 1 is the non-designated supplier. The long-term stake corresponds to the difference for that supplier between collusive and competitive profits in all future tenders, evaluated at period  $t$ . The next tender (for the other market, where the stake for supplier 1 is thus equal to  $\bar{\pi}^m - \bar{\pi}^c$ ) comes in period  $t + \tau - 1$  and the following one (for the same market as the current one, where the stake for supplier 1 is thus equal to  $\bar{\pi}^n - \bar{\pi}^c$ ) in period  $t + T$ ; as tenders occur

every  $T$  periods for each market, the long-term stake for supplier 1 is thus equal to:

$$\begin{aligned} L_1(\delta; T, \tau) &= \delta^{\tau-1} (\bar{\pi}^m - \bar{\pi}^c) (1 + \delta^T + \dots) + \delta^T (\bar{\pi}^n - \bar{\pi}^c) (1 + \delta^T + \dots) \\ &= \frac{\delta^{\tau-1}}{1 - \delta^T} (\bar{\pi}^m - \bar{\pi}^c) - \frac{\delta^T}{1 - \delta^T} (\bar{\pi}^c - \bar{\pi}^n). \end{aligned}$$

Consider now the tender for the other market taking place in period  $t + \tau$ , which is assigned to supplier 1. From the standpoint of supplier 2, the next tender comes in period  $t + T + 1$ , and the following one in period  $t + \tau + T$ . The long-term stake for supplier 2 is thus equal to:

$$L_2(\delta; T, \tau) = \frac{\delta^{T-\tau+1}}{1 - \delta^T} (\bar{\pi}^m - \bar{\pi}^c) - \frac{\delta^T}{1 - \delta^T} (\bar{\pi}^c - \bar{\pi}^n).$$

The conditions for sustainability and initiation are more stringent for lower values of the long-term stake, and thus the relevant stake is  $\min \{L_1(\delta; T, \tau), L_2(\delta; T, \tau)\}$ , which is given by (21). Note that this expression indeed coincides with  $L^{Sync}(\delta^{T/2})$  for  $\tau = 1$ , and so is valid for the entire range  $\tau \in \{1, \dots, T\}$ .

It is straightforward to check that, keeping  $T$  fixed, the right side of (21) decreases as  $\alpha(\cdot, \tau)$  increases. Hence, collusion is easiest for the lowest value of  $\alpha(t, \tau)$ , which is obtained for  $\tau = T/2$  if  $T$  is an even number, and for  $\tau \in \{(T+1)/2, (T+3)/2\}$  otherwise. By contrast, collusion is the most difficult for the lowest value of  $\alpha(T, \tau)$ , which is achieved for  $\tau = 1$ , that is, for perfectly synchronous purchasing.

Finally, we check that collusion is more difficult as the length of the contracts increases. This is obvious in the case of synchronous purchasing, as

$$L^{Sync}(\delta^{T/2}) = \frac{\delta^T}{1 - \delta^T} (\bar{\pi}^m + \bar{\pi}^n - 2\bar{\pi}^c)$$

decreases as  $T$  increases (recall that we focus on markets that are at risk, implying that the bracketed expression is positive). For the case of perfectly staggered purchasing, the long-term stake is given by (and ignoring integer problems for the sake of exposition):

$$L^{Stag}(\delta; T) = L_1(\delta; \frac{T}{2} + 1, \tau) = L_2(\delta; \frac{T}{2} + 1, \tau) = \frac{\delta^{\frac{T}{2}}}{1 - \delta^T} (\bar{\pi}^m - \bar{\pi}^c) - \frac{\delta^T}{1 - \delta^T} (\bar{\pi}^c - \bar{\pi}^n),$$

which, using  $x \equiv \delta^T$  and  $\alpha \equiv (\bar{\pi}^m - \bar{\pi}^c) / (\bar{\pi}^c - \bar{\pi}^n)$ , can be expressed as:

$$L^{Stag}(\delta; T) = \phi(x) \equiv \frac{x(1 - \alpha x)}{1 - x^2}.$$

We have:

$$\phi'(x) = \frac{d}{dx} \left( \frac{x(1-\alpha x)}{1-x^2} \right) = \frac{1-2\alpha x+x^2}{(1-x^2)^2} > \frac{1-2x+x^2}{(1-x^2)^2} = \frac{1}{(1+x)^2} > 0,$$

where the first inequality follows from the fact that the market is at risk, implying  $\alpha < 1$ . It follows that the long-term stake increases with  $x = \delta^T$ , and thus decreases as  $T$  increases. ■

## A.6 Proof of Proposition 8

To start, note that for  $c \in (\underline{c}, \bar{c}]$ ,

$$\frac{\partial G_\sigma(c)}{\partial \sigma} = -[1 - G(c)]^\sigma \ln[1 - G(c)] > 0, \quad (26)$$

and

$$\lim_{\sigma \rightarrow \infty} G_\sigma(c) = 1. \quad (27)$$

Under our assumption that  $r = \bar{c}$ , we have  $\bar{\pi}^n = 0$ ,  $S_{SPA} = \pi^c(\underline{c})$ , and  $S_{FPA} = \bar{c} - \underline{c}$ . Because

$$\pi^c(\underline{c}) = \mathbb{E}_{c_{-i}} [\max\{0, c_{-i} - \underline{c}\}] = \bar{c} - \underline{c} - \int_{\underline{c}}^{\bar{c}} G_\sigma(x) dx,$$

it follows that

$$\frac{\partial \pi^c(\underline{c})}{\partial \sigma} = - \int_{\underline{c}}^{\bar{c}} \frac{\partial G_\sigma(x)}{\partial \sigma} dx < 0,$$

so the competitive payoff, and hence  $\pi_{SPA}^d(\underline{c})$ , is decreasing in  $\sigma$ . Further, for all  $\lambda \in [0, 1]$ ,

$$\begin{aligned} \frac{\partial}{\partial \sigma} (\bar{\pi}^m - (1+\lambda)\bar{\pi}^c) &= \frac{\partial}{\partial \sigma} \left( \int_{\underline{c}}^{\bar{c}} G_\sigma(c) dc - (1+\lambda) \int_{\underline{c}}^{\bar{c}} G_\sigma(c)(1-G_\sigma(c)) dc \right) \\ &= \int_{\underline{c}}^{\bar{c}} \frac{\partial G_\sigma(c)}{\partial \sigma} [2(1+\lambda)G_\sigma(c) - \lambda] dc, \end{aligned}$$

which is positive for  $\sigma$  sufficiently large by (26) and (27). Thus, considering synchronized purchasing, the right side of (13) has a numerator that is either constant or decreasing in  $\sigma$  and, based on the analysis above with  $\lambda = 1$ , a denominator that is increasing in  $\sigma$  for  $\sigma$  sufficiently large. Because the numerator and denominator are positive, it follows that for  $\sigma$  sufficiently large, the threshold discount factor is decreasing in  $\sigma$ .

Turning to the case of staggered purchasing, and totally differentiating (14) with

respect to  $\sigma$ , we have

$$\frac{\partial \delta}{\partial \sigma} = \frac{1}{\Delta} \frac{\frac{\partial \pi_a^d(\underline{c})}{\partial \sigma} (\bar{\pi}^m - (1 + \delta)\bar{\pi}^c) - \pi_a^d(\underline{c}) \left[ \frac{\partial \bar{\pi}^m}{\partial \sigma} - (1 + \delta) \frac{\partial \bar{\pi}^c}{\partial \sigma} \right]}{(\bar{\pi}^m - (1 + \delta)\bar{\pi}^c)^2}, \quad (28)$$

where

$$\Delta = \frac{1 + \delta^2}{1 - \delta^2} - \frac{\bar{\pi}^c}{[\bar{\pi}^m - (1 + \delta)\bar{\pi}^c]^2}.$$

For  $\sigma$  sufficiently large,  $\bar{\pi}^c$  is close to zero and  $\bar{\pi}^m$  is close to  $\bar{c} - \underline{c}$ , so  $\Delta > 0$ . Further, by the arguments above the numerator in (28) is negative for  $\sigma$  sufficiently large. Thus, we conclude that  $\frac{\partial \delta}{\partial \sigma} < 0$ , which completes the proof. ■

## B Deterrence with asymmetric reserves for staggered purchasing

In this appendix, we analyze optimal reserves under staggered purchasing for a single buyer setting possibly different reserves in the two markets. As in Section 5.5, we assume costs are uniformly distributed and that  $v = 1$ , which ensures the monotonicity of the long-term stake with respect to  $\delta$ .

Under staggered purchasing, there are three relevant timing possibilities as depicted below, where we denote the suppliers by  $A$  and  $B$ :

Market allocation					
Period	1	2	3	4	...
Active market	1	2	1	2	...
Designated supplier	A	B	A	B	...

Rotation 1						Rotation 2					
Period	1	2	3	4	...	Period	1	2	3	4	...
Active market	1	2	1	2	...	Active market	1	2	1	2	...
Designated supplier	A	A	B	B	...	Designated supplier	A	B	B	A	...

The key difference between the two rotations is that in rotation 1, the suppliers begin their two consecutive periods of being designated with market 1, whereas in rotation 2, the suppliers begin their two consecutive periods of being designated with market 2.

Under a market allocation, the long-term stake for a supplier that will be designated in market 1 next period is

$$L^{Stag}(\delta, r_1, r_2) \equiv \frac{\delta}{1 - \delta^2} (\bar{\pi}^m(r_1) - \bar{\pi}^c(r_1)) - \frac{\delta^2}{1 - \delta^2} (\bar{\pi}^c(r_2) - \bar{\pi}^n(r_2)),$$

and the long-term stake for a supplier that will be designated in market 2 next period is  $L^{Stag}(\delta, r_2, r_1)$ .

The long-term stake for a supplier that is not designated for either market, but that will rotate to market 1 next period, then market 2, and then “sit out” for two periods is

$$\begin{aligned} L_{Rot}^{Stag}(\delta, r_1, r_2) \equiv & \frac{\delta}{1 - \delta^4} (\bar{\pi}^m(r_1) - \bar{\pi}^c(r_1)) + \frac{\delta^2}{1 - \delta^4} (\bar{\pi}^m(r_2) - \bar{\pi}^c(r_2)) \\ & - \frac{\delta^3}{1 - \delta^4} (\bar{\pi}^c(r_1) - \bar{\pi}^n(r_1)) - \frac{\delta^4}{1 - \delta^4} (\bar{\pi}^c(r_2) - \bar{\pi}^n(r_2)), \end{aligned}$$

and the long-term stake for a supplier that is not designated for either market, and that will also not be designated for either market next period, but that will rotate to market 1 in the following period, then market 2, and then “sit out” for two periods is

$$\begin{aligned} \hat{L}_{Rot}^{Stag}(\delta, r_1, r_2) \equiv & -\frac{\delta}{1 - \delta^4} (\bar{\pi}^c(r_2) - \bar{\pi}^n(r_2)) + \frac{\delta^2}{1 - \delta^4} (\bar{\pi}^m(r_1) - \bar{\pi}^c(r_1)) \\ & + \frac{\delta^3}{1 - \delta^4} (\bar{\pi}^m(r_2) - \bar{\pi}^c(r_2)) - \frac{\delta^4}{1 - \delta^4} (\bar{\pi}^c(r_1) - \bar{\pi}^n(r_1)). \end{aligned}$$

Analogously, the long-term stake for a supplier that is not designated for either market, but that will rotate to market 2 next period, then market 1, and then “sit out” for two periods is  $L_{Rot}^{Stag}(\delta, r_2, r_1)$ . The long-term stake for a supplier that is not designated for either market, and that will also not be designated for either market next period, but that will rotate to market 2 in the following period, then market 1, and then “sit out” for two periods is  $\hat{L}_{Rot}^{Stag}(\delta, r_2, r_1)$ . As in the case of synchronized purchasing, a rotation involving both markets is easier to sustain than one involving only one market for discount factors sufficiently large, including all discount factors in the relevant range when costs are uniformly distributed. We assume that colluding suppliers use either a market allocation or a rotation involving both markets.

For staggered purchasing, a market allocation is sustainable if

$$L^{Stag}(\delta, r_1, r_2) \geq S_a(r_2) \quad \text{and} \quad L^{Stag}(\delta, r_2, r_1) \geq S_a(r_1), \quad (29)$$

and a rotation is sustainable if either

$$L_{Rot}^{Stag}(\delta, r_1, r_2) \geq S_a(r_2) \quad \text{and} \quad \hat{L}_{Rot}^{Stag}(\delta, r_1, r_2) \geq S_a(r_1) \quad (30)$$

or

$$L_{Rot}^{Stag}(\delta, r_2, r_1) \geq S_a(r_1) \quad \text{and} \quad \hat{L}_{Rot}^{Stag}(\delta, r_2, r_1) \geq S_a(r_2). \quad (31)$$

As in the case of synchronized purchasing, there exists a largest discount factor  $\underline{\delta}_a^{Stag}(v)$  such that there is competition at reserves of  $r_1 = r_2 = r^{Comp}(v)$ . For  $\delta > \underline{\delta}_a^{Stag}(v)$ , define

$$(r_{1,a}^{Stag}(\delta), r_{2,a}^{Stag}(\delta)) \in \arg \max_{(r_1, r_2) \in [\underline{c}, \min\{\bar{c}, v\}]^2} \{U^{Comp}(r_1) + U^{Comp}(r_2) \mid r_1 \leq r_2, (29)-(31) \text{ all fail}\},$$

and define  $\bar{\delta}_a^{Stag}(v)$  to be the largest discount factor such that deterrence is optimal:

$$\begin{aligned} \bar{\delta}_a^{Stag}(v) &\equiv \max \left\{ \delta \in [0, 1] \mid U^{Comp}(r_{1,a}^{Stag}(\delta)) + U^{Comp}(r_{2,a}^{Stag}(\delta)) \geq 2U^{Coll}(r^{Coll}(v)) \right\} \\ &= \min \left\{ \delta \in [0, 1] \mid U^{Comp}(r_{1,a}^{Stag}(\delta)) + U^{Comp}(r_{2,a}^{Stag}(\delta)) \leq 2U^{Coll}(r^{Coll}(v)) \right\}, \end{aligned}$$

where, as with synchronized purchasing, the equality uses the monotonicity of the long-term stake in  $\delta$ . Thus, a characterization of optimal reserves analogous to (25) holds for the case of staggered purchasing.

Although the qualitative results is the same as for synchronized purchasing, characterization of supplier conduct as a function of the reserves and the optimal reserves look a bit different than for synchronized purchasing because for sufficiently symmetric reserves, a market allocation is easier for the suppliers to sustain, but for asymmetric reserves, a rotation is easier for the suppliers to sustain. For intuition, note that in a market allocation, a deviator is always the designated supplier in the next period, but in a rotation, a supplier might deviate when it is not the designated supplier in the next period (i.e., deviate in the first of the two consecutive periods when the supplier is not designated), which means the long-term stake is lower. This means that for symmetric reserves, a market allocation is easier to sustain. But for sufficiently asymmetric reserves, the market allocation is harder to support because the long-term stake of the supplier in the market with the low reserve is low, while the short-term stake in the market with the high reserve is high.

We illustrate the optimal reserves under staggered purchasing in Figure 8. As shown in Figure 8(a), supplier conduct may involve either market allocation or rotation, depending on reserves. When deterrence is optimal, as shown in Figure 8(b), the optimal deterrence reserves are asymmetric and, as the discount factor increases, initially leave the suppliers indifferent between competition and market allocation, but then for higher discount

factors, the relevant constraints are those that relate to the suppliers not switching to a rotation.

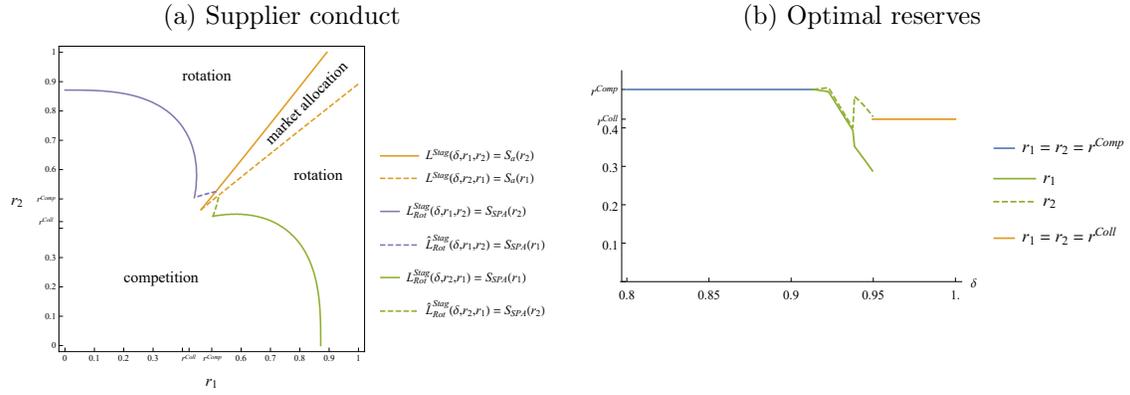


Figure 8: Illustration of supplier conduct and optimal reserves under staggered purchasing and second-price auctions. Assumes that costs are uniformly distributed over  $[0, 1]$ ,  $v = 1$ , and  $\delta = 0.92$ .

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